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**ORIGINAL ARTICLE**

# MHD Visco-Elastic Boundary Layer Flow through Porous Medium with Chemical Reaction

**Narendra Singh[1] and U.R. Singh[2]**
[1]Rani Shanti Devi Mahavidyalaya, Hathaundha, Ram Sanehi Ghat, Barabanki
[2]C.L. Jain (PG) College, Firozabad
Email: nrnarendrarajput@gmail.com

## ABSTRACT

*This section concerned with the behavior of the influence of a first-order homogeneous chemical reaction on free convective boundary layer flow of an electrically conducting visco-elastic, incompressible fluid through porous medium over a continuously moving flat surface in the presence of uniform magnetic field and constant suction is studied. A uniform magnetic field is assumed to be applied perpendicular to the moving flat surface. The velocity, temperature and concentration field are obtained. The effect of various parameters on the velocity, temperature and concentration profile as discussed graphically.*

***Key Words:*** *Magneto hydrodynamics, visco-elastic, porous media, boundary layer flow, free convection, heat and mass transfer, chemical reaction*

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## INTRODUCTION

From a technological point of view, a study of boundary layer flow on continuously moving surface is always important. The analysis of such flows has applications in different areas such as aerodynamic extrusion of plastics sheets, boundary layer along material handling conveyors, cooling of an infinite metallic plate in a cooling bath and boundary layer along a liquid in condensation process. In view of these applications Soundalgekar (1977) had studied free convection effects on the stokes problem for an infinite vertical plate; Abdelhafez (1985) discussed the skin friction and heat transfer on a continuous flat surface moving in a parallel free stream; Vajravelu (1988) worked on a singular perturbation solution for a hydromagnetic flow; Singh (1991) has studied the effects of hydromagnetic convection flow past a porous plate; Khan *et. al.,* (1998) have discussed MHD free convection boundary layer flow through porous media; further khan (1999) discussed the MHD free convection past a continuous moving surface; Choudhary and Das (2000) have worked on magneto hydrodynamics boundary layer flows of non-Newtonian fluid past a flat plate; Jain *et. al.,* (2003) have studied the free convection and hydromagnetic effects on the three dimensional flow past an accelerated porous plate; Ahmad *et. al.,* (2004) have studied the Hall effects on the free convection flow of a non-Newtonian power law fluid at a stretching surface. Kumar and Singh (2008) have discussed on MHD vico-elastic boundary layer flow through porous medium with free convection past a continuous moving surface. Recently, Bhagwat and Kuldeep (2010) have studied on effect of mass transfer on MHD visco-elastic boundary layer flow through porous medium with free convection past a continuous moving plate.

In section, our aim is to study the effects of chemical reaction on free convection on boundary layer flow of an electrically conducting visco-elastic incompressible fluid through porous medium over a continuously moving flat surface in the presence of a uniform magnetic field.

## FORMULATION OF THE PROBLEM

Consider a long continuous sheet which issues a slot. Let us take the assumptions that a certain time has elapsed after initiation of motion, so that the steady state conditions prevail and the flow disturbances created by the roll are neglected. An observer fixed in space will note that the boundary layer on the sheet originates at the slot and grows in the direction of the sheet. The boundary behavior here appears to be different form what would be expected, if sheet were considered as moving flat plate of finite length, on which the boundary would grow in the direction opposite to the direction of motion of the plate.
Let us consider a first-order homogeneous chemical reaction on boundary layer flow of an electrically conducting incompressible, visco-elastic fluid (Walter's Liquid-B) over a continuous moving flat surface through porous medium and $B_0$ as imposed uniform magnetic field perpendicular to the surface. Let us denote velocity components $U$, $V$ in direction $X$ and $Y$ respectively temperature and concentration denoted by $T$ & $C$ under these assumptions the physical variables are functions of $Y$ only.
The governing equations are:-

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad \text{...(1)}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = g\beta\,(T - T_\infty) + g\beta^*(C - C_\infty)$$

$$+ \nu\frac{\partial^2 U}{\partial Y^2} - \lambda_0^*\left[U\frac{\partial^3 U}{\partial X\partial Y^2} + V\frac{\partial^2 U}{\partial Y^2} + \frac{\partial U}{\partial X}.\frac{\partial^2 U}{\partial Y^2}\right] - \frac{\sigma B_0^2}{\rho}U - \frac{\mu}{K^*}U \quad \text{...(2)}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \alpha\frac{\partial^2 T}{\partial Y^2} \quad \text{...(3)}$$

$$U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = D\frac{\partial^2 C}{\partial Y^2} - K_r C \quad \text{...(4)}$$

Where,

$\rho$ = The density
$\nu$ = The Kinematic viscosity
$\sigma$ = The Electric Conductivity
$B_0$ = The Magnetic Induction
$g$ = Acceleration due to gravity
$\beta$ = Coefficient of Volume Expansion
$T$ = The temperature of the fluid in the free stream
$\alpha$ = The Thermal Diffusivity
$\lambda_0^*$ = The Visco-elastic parameter
$\mu$ = Coefficient of Viscosity
$K^*$ = Permeability Parameter
$K_r$ = Chemical reaction parameter
$V_0$ = Suction Velocity
$C$ = The concentration of the fluid in the free stream
$D$ = The mass diffusion coefficient

Subject to boundary conditions

$$\left.\begin{aligned} &U = U_w, \quad V = -V_0 = cons., \quad T = T_{w,} \quad C = C_w \quad at \quad y = 0 \\ &U \to 0, \quad T \to T_{w,} \quad C \to C_w \qquad as \qquad y \to 0 \end{aligned}\right\} \quad ...(5)$$

Making use of the assumption that the velocity field is independent of distance parallel to the surface, equations (1) to (4) and boundary conditions (5) can be written as

$$-V_0 \frac{dU}{dY} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty)$$

$$+\upsilon \frac{d^2U}{dY^2} + \lambda_0^* \frac{d^3U}{dY^3} - \frac{\sigma B_0^2}{\rho}U - \frac{\mu}{K^*}U \quad ...(6)$$

$$-V_0 \frac{dT}{dY} = \alpha \frac{d^2T}{dY^2} \quad ...(7)$$

$$-V_0 \frac{dC}{dY} = D\frac{d^2C}{dY^2} - K_r C \quad ...(8)$$

Boundary conditions are

$$\left.\begin{aligned} &U = U_w, \quad T = T_{w,} \quad C = C_w \quad at \quad y = 0 \\ &U \to 0, \quad T \to T_{w,} \quad C \to C_w \quad as \quad y \to 0 \end{aligned}\right\} \quad ...(9)$$

On introducing the following non-dimensional quantities

$$y = \frac{Y}{L}, \quad u = \frac{U}{U_w}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad K = \frac{K^* U_w^2}{\upsilon^2}$$

Where $L$ is the characteristic length (between the slit and wind up roll)

$$R = \frac{V_0 L}{\upsilon} \text{ (Reynolds number)}$$

$$\Pr = \frac{\upsilon}{\alpha} \text{ (Prandtl number)}$$

$$Sc = \frac{\upsilon}{D} \text{ (Schmidt number)}$$

$$\lambda_0 = \frac{\lambda_0^*}{L^2} \text{ (viscoelastic parameter)}$$

$$Kr = \frac{K' r\upsilon}{V_0^2} \text{ (Chemical reaction parameter)}$$

$$M = \frac{\sigma B_0^2 L}{\rho V_0} \text{ (Hartman number)}$$

$$Gr = \frac{g\beta(T_w - T_\infty)L}{V_0 U_w} \text{ (Grashoff number)}$$

$$Gm = \frac{g\beta^*(C_w - C_\infty)L}{V_0 U_w} \text{ (Modified Grashoff number)}$$

$$\frac{1}{K} = \frac{\rho U_w^2 L}{K^* \upsilon}, \qquad M_1 = \left(M + \frac{1}{K}\right)$$

Now, equations (2.6) to (2.8) in non-dimensional form are

$$\lambda_0 \frac{d^3u}{dy^3} + \frac{1}{R}\frac{d^2u}{dy^2} + \frac{du}{dy} - M_1 u = -(Gr\theta + Gm\phi) \quad ...(10)$$

$$\frac{d^2\theta}{dy^2} + R\Pr\frac{d\theta}{dy} = 0 \quad ...(11)$$

$$\frac{d^2\phi}{dy^2} + R\ Sc\frac{d\phi}{dy} - ScK_r\phi = 0 \quad ...(12)$$

And the boundary conditions reduces to

$$\left.\begin{array}{lllll} u = 1, & \theta = 1, & \phi = 1 & at & y = 0 \\ u \to 0, & \theta \to 0, & \phi \to 0 & as & y \to 0 \end{array}\right\} \quad ...(13)$$

Solving equations (10) to (12) under the boundary conditions (13), we get

$$u = e^{-my} + GrA_1(e^{-ny} - e^{-my}) + GmA_2(e^{-ly} - e^{-my}) \quad ...(14)$$

$$\theta = e^{-ny} \quad ...(15)$$

$$\phi = e^{-ly} \quad ...(16)$$

where *m* is the root of cubic equation by numerical methods.

$$\lambda_0 r^3 + \frac{1}{R}r^2 + r - M_1 = 0 \quad ...(17)$$

Where,

$$n = (R\Pr)$$

$$l = \frac{1}{2}\left(RSc + \sqrt{R^2Sc^2 + 4K_rSc}\right)$$

$$A_1 = \frac{1}{\lambda_0 n^3 - n\left(\frac{n}{R} - 1\right) + M_1}$$

$$A_1 = \frac{1}{\lambda_0 l^3 - l\left(\frac{l}{R} - 1\right) + M_1}$$

## SKIN FRICTION

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -m + GrA_1(m-n) + GmA_2(m-RSc) \quad ...(18)$$

The rate of heat transfer =- $\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = n$ ...(19)

The rate of mass transfer =- $\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = l$ ...(20)

## RESULTS AND DISCUSSION

The velocity, temperature and concentration profiles of boundary layer flow are plotted in figure 1-14 for different values of small Reynolds number ®, Hartman number (*M*), Grashoff number (*Gr*), modified Grashoff number (*Gm*), visco-elastic parameter ($\lambda_0$), permeability parameter (*K*), Prandtl number (Pr), chemical reaction parameter ($K_r$) and Schmidt number (*Sc*).

Figure 1-9 represent the velocity profiles of boundary layer flow for different parameters. Figure- 1 shows the variation of velocity *u* with magnetic parameter *M*. It is observed that the velocity decreases as *M* increases. Figure- 2 shows that an increase in permeability parameter *K* causes an increase in velocity profile of boundary layer flow. From Figure-3, it is observed that the velocity of boundary layer flow increases as the Grashoff number *Gr* increase. The variation of *u* with modified Grashoff number *Gm* is shown in Figure- 4. It is noticed that increase in *Gm* leads to increase in velocity of boundary layer flow. From Figure- 5 shows the variation of velocity *u* with Reynolds number ®. It is observed that the velocity of boundary layer flow decreases as *R* increases. In figure- 6, the velocity profile of boundary layer flow increases due to increasing visco-elastic parameter ($\lambda_0$). Figure- 7, shows the variation of velocity profile of boundary layer flow (*u*) with Prandtl number (Pr). It is observed that the velocity decreases as Pr increases. From figure- 8, shows the variation of velocity profile of boundary layer flow (*u*) with Schmidt number (*Sc*). It is observed that the velocity of boundary layer flow (*u*) decreases as *Sc* increases. The velocity profile of boundary layer flow for Chemical 101eaction parameter ($K_r$) is shown in figure- 9. It is clear that velocity of boundary layer flow *u* decreases with increasing in $K_r$.

Figures 10-13 represent the temperature profiles of boundary layer flow for different parameters. The temperature profiles of boundary layer flow for Prandtl number (Pr) is shown in figure- 10. It is observed that increase in Prandtl number Pr causes decrease in temperature profile of boundary layer flow. From figure- 11 shows the variation of temperature $\theta$ with Reynolds number *R*. It is observed that the temperature profile of boundary layer flow decreases as *R* increases.

Figure 12-14 represent the concentration profiles of boundary layer flow for different parameters. From Figure- 12, it is noticed that an increase in Schmidt number *Sc* leads to decrease in concentration profile of boundary layer flow. Figure- 13 shows that an increase in Reynolds number ® causes decrease in concentration profile of boundary layer flow. The concentration profiles of boundary layer flow for Chemical Reaction parameter ($K_r$) is shown in figure- 14. It is clear that concentration profiles of boundary layer flow increases with increasing in $K_r$.

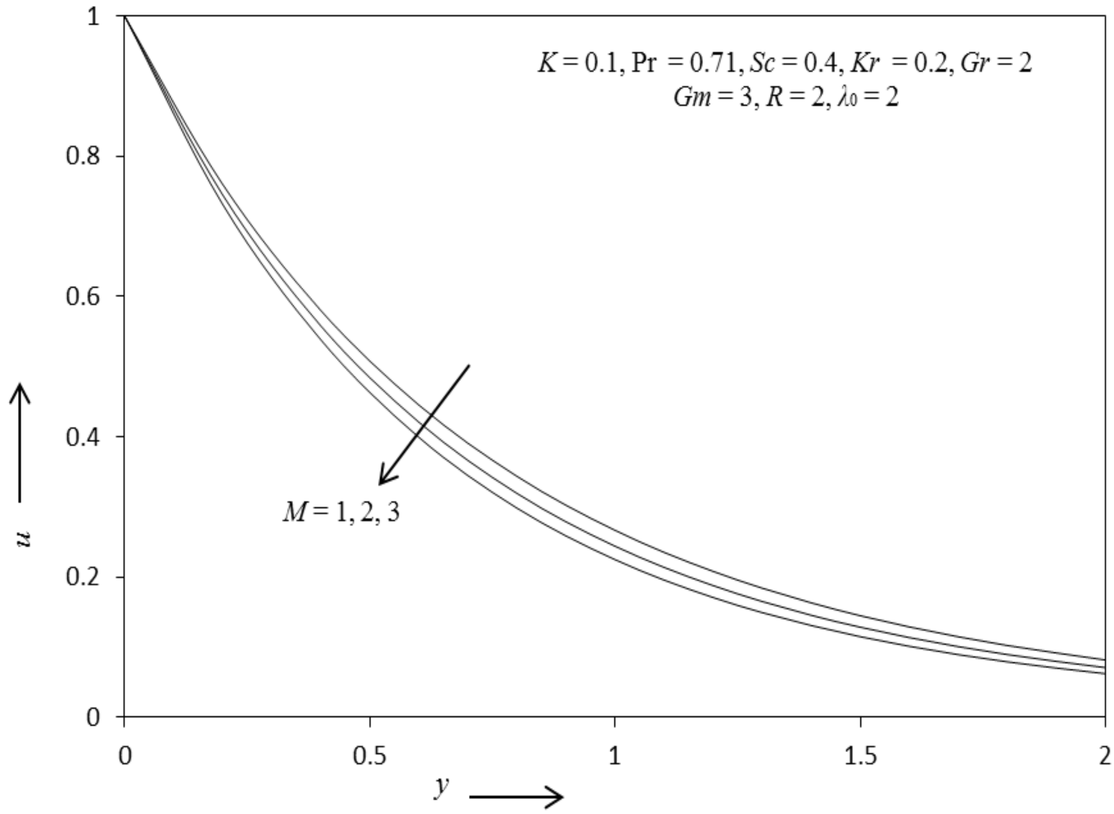


Fig.- (1): Velocity profile of fluid for different values of $M$.

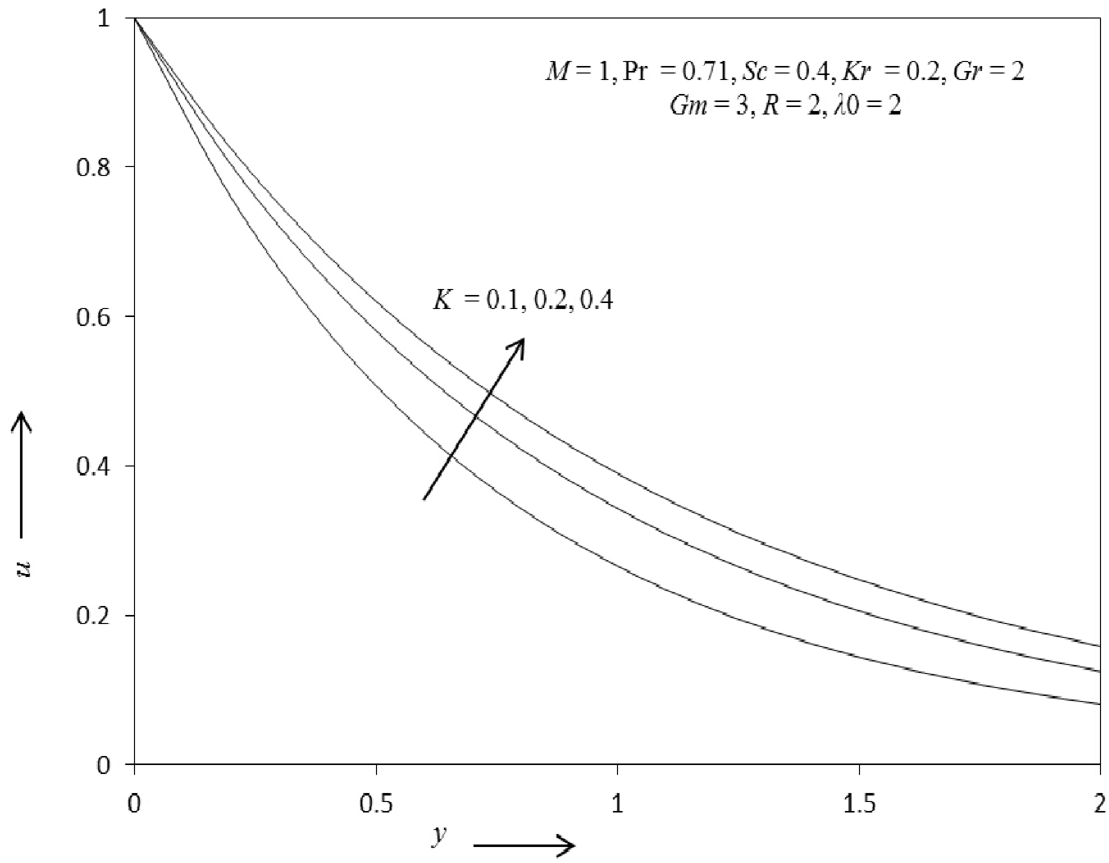


Fig.- (2): Velocity profile of fluid for different values of $K$.

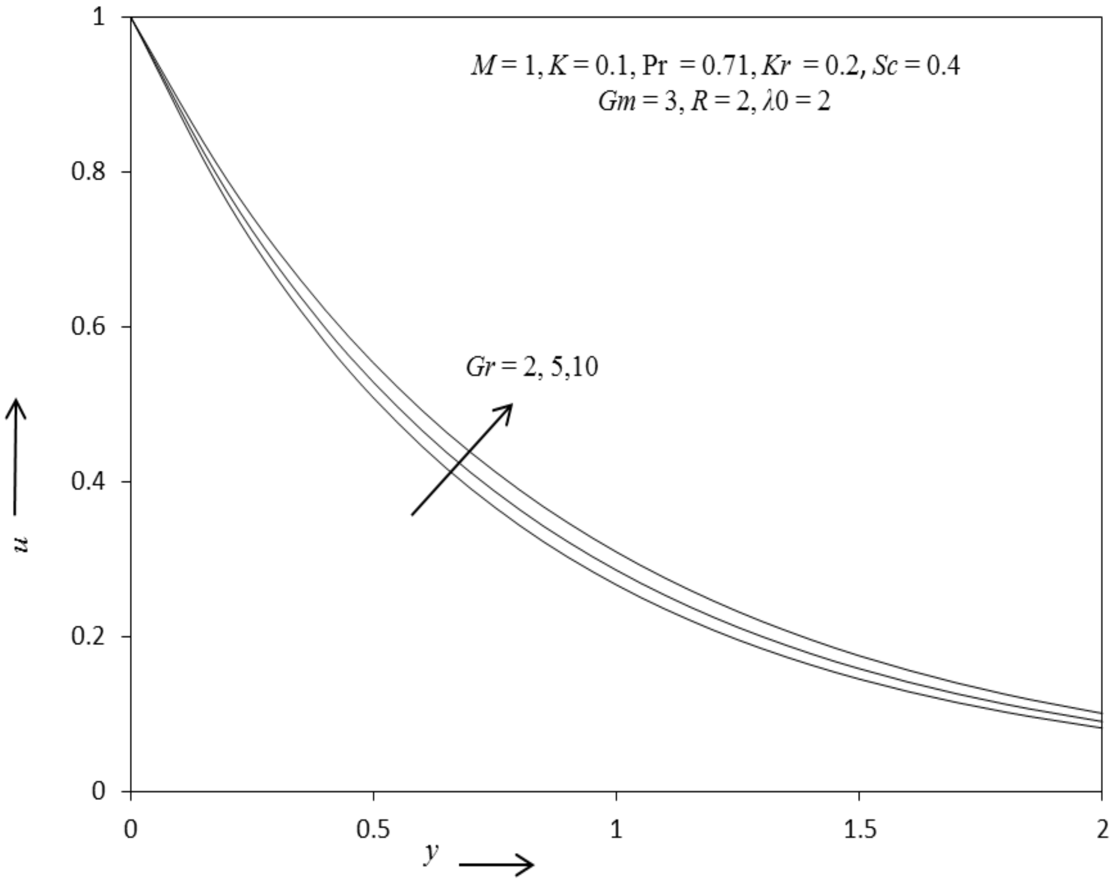


Fig.- (3): Velocity profile of fluid for different values of *Gr*.

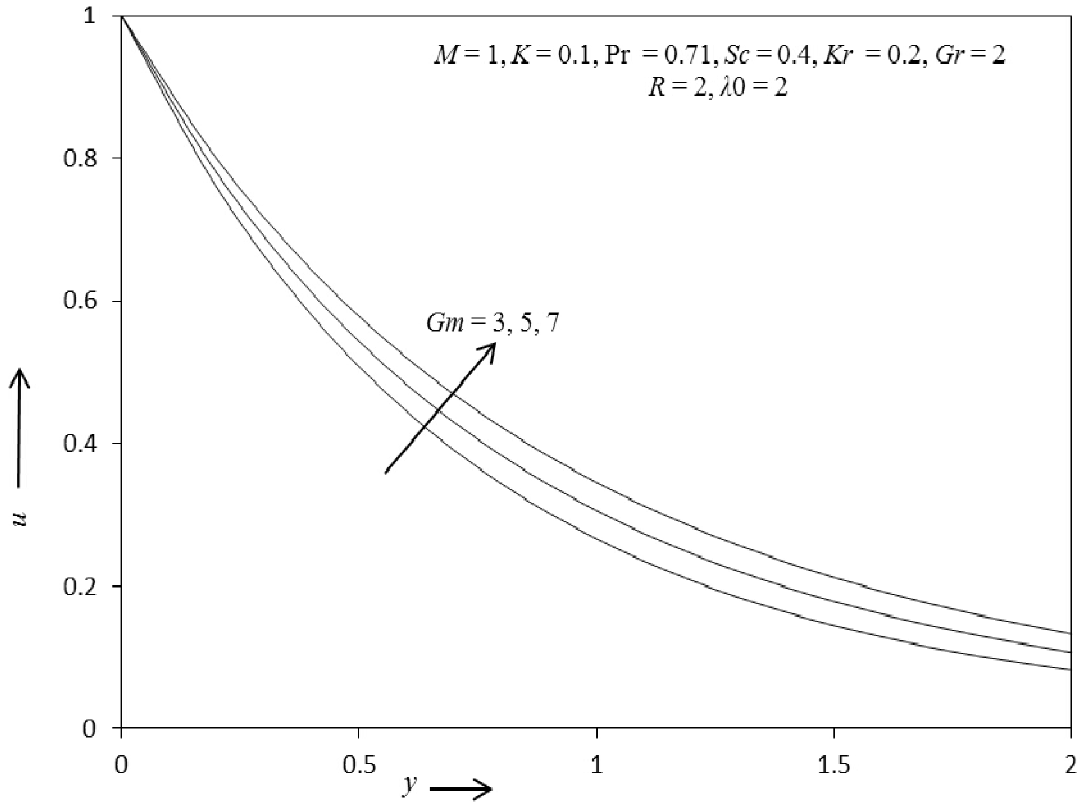


Fig.- (4): Velocity profile of fluid for different values of *Gm*.

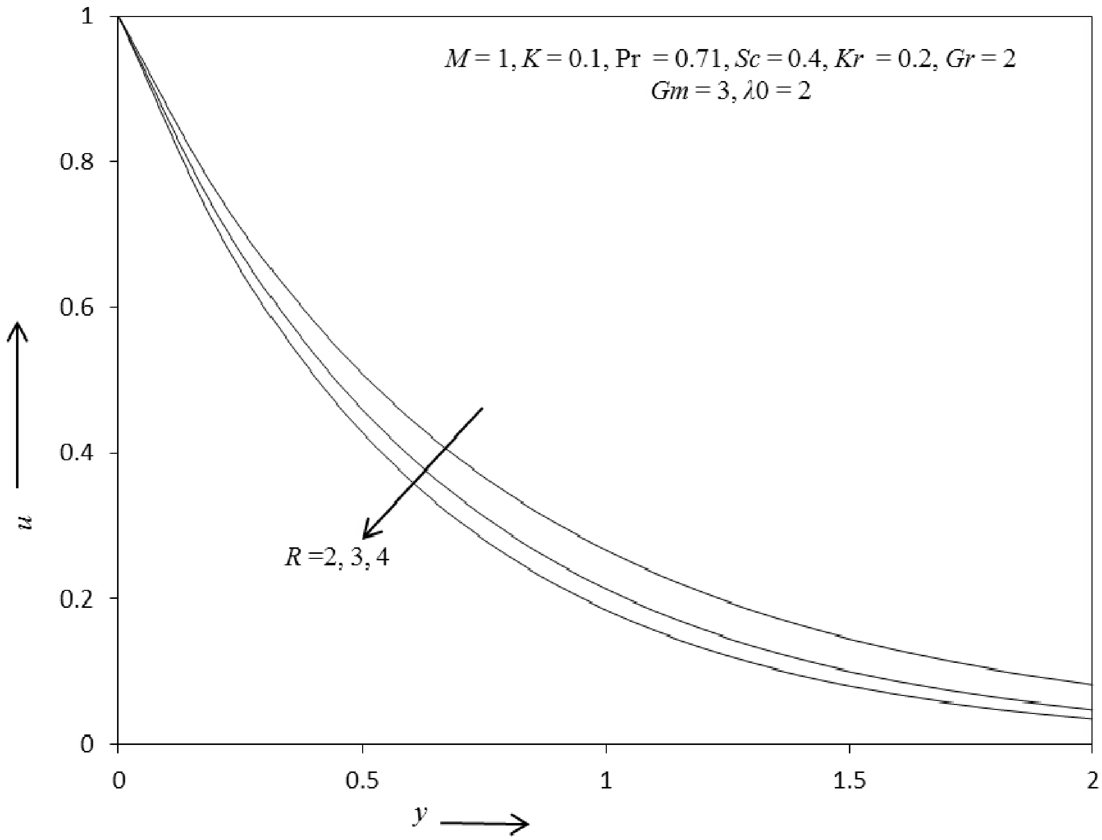


Fig.- (5): Velocity profile of fluid for different values of $R$.

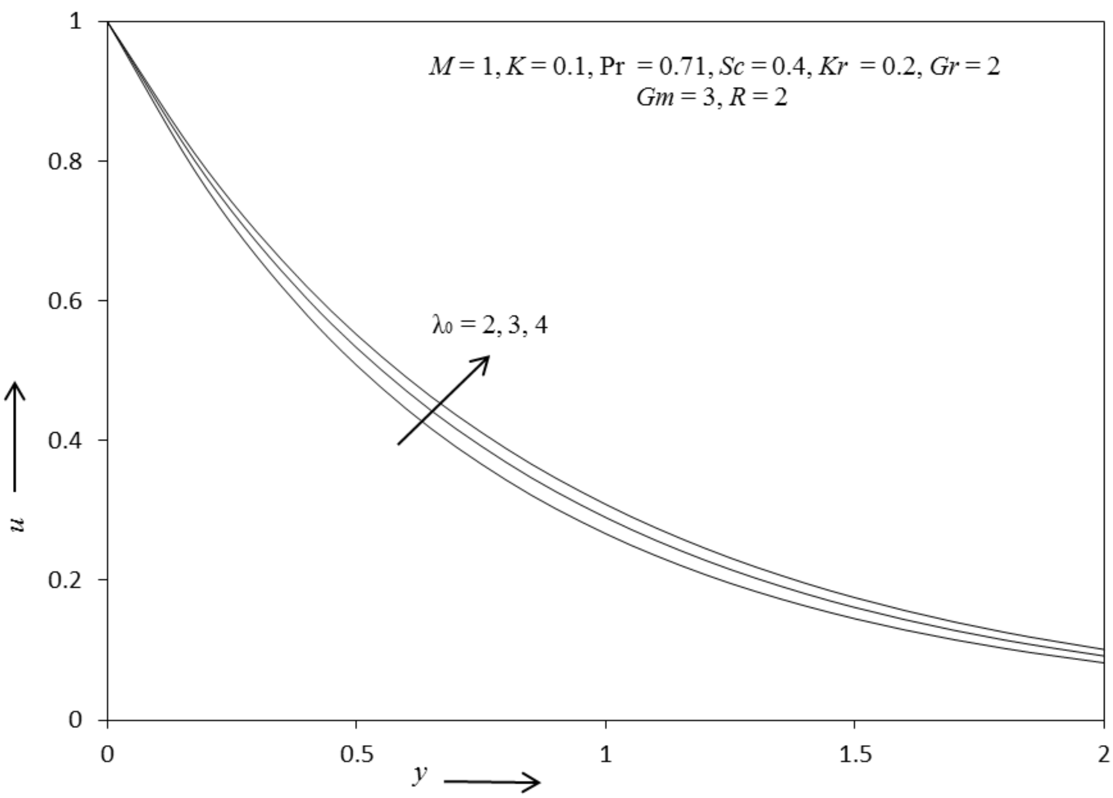


Fig.- (6): Velocity profile of fluid for different values of $\lambda_0$.

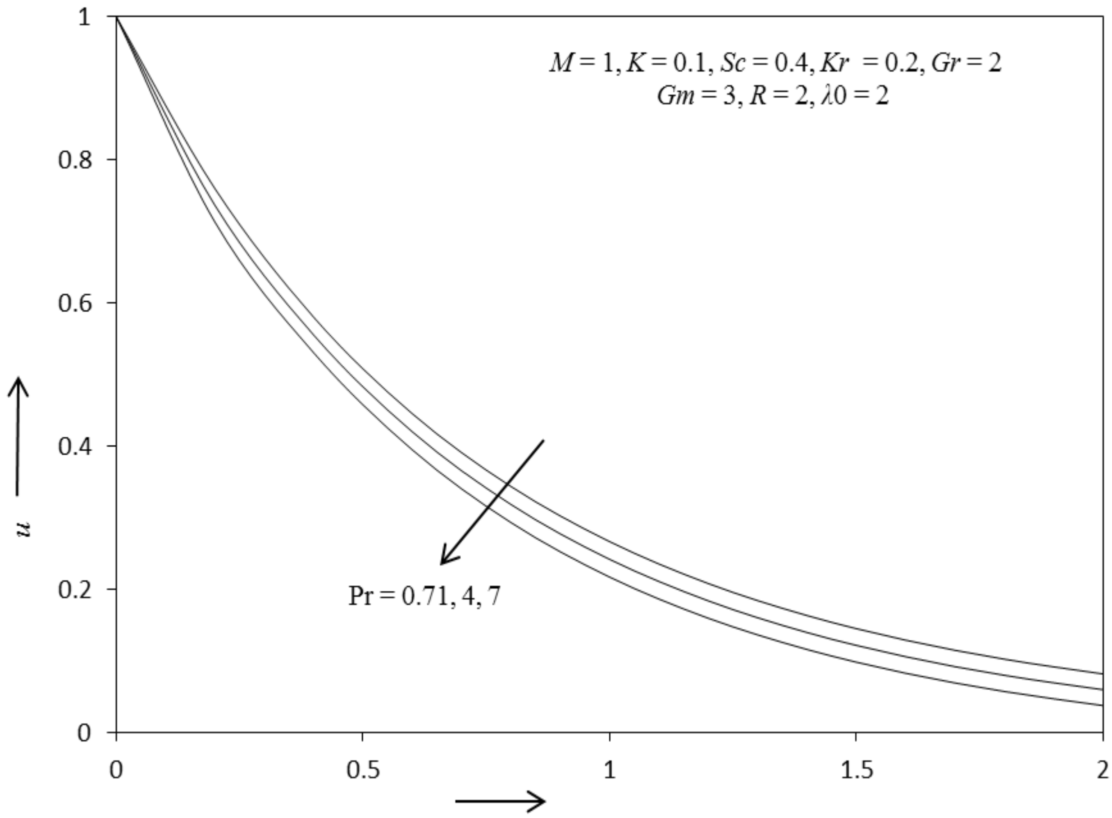


Fig.- (7): Velocity profile of fluid for different values of Pr.

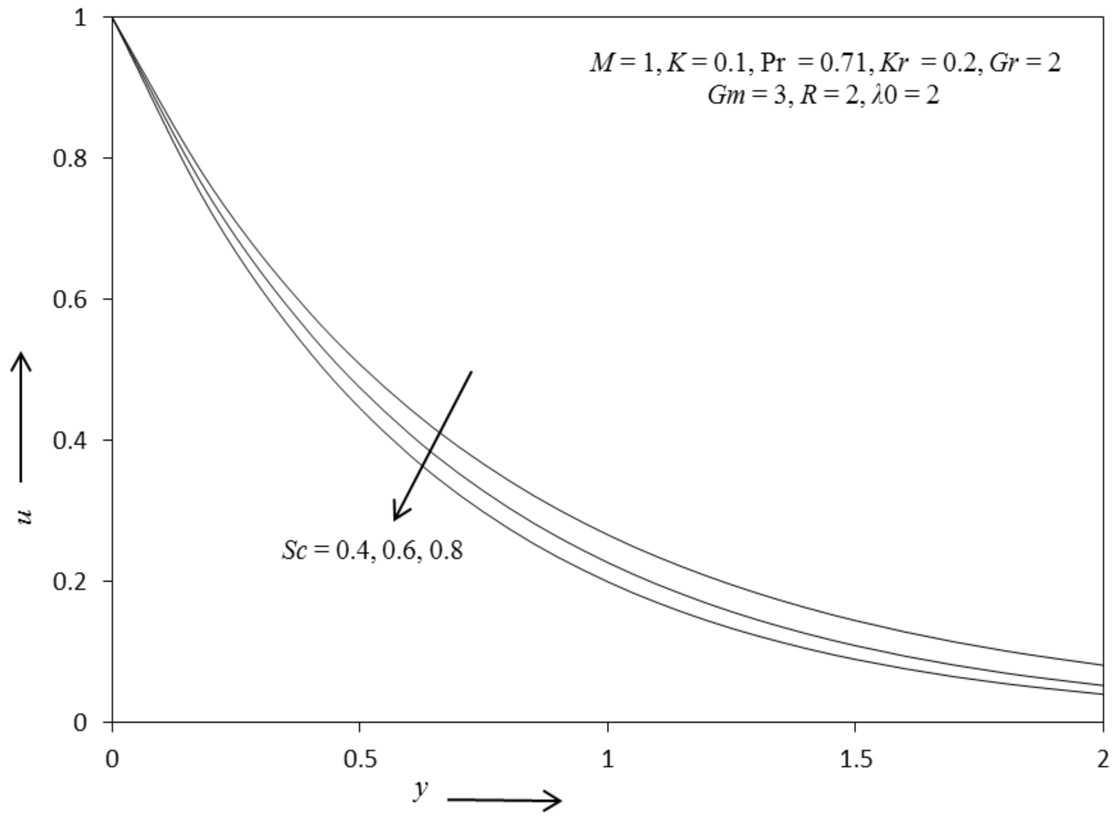


Fig.- (8): Velocity profile of fluid for different values of *Sc*.

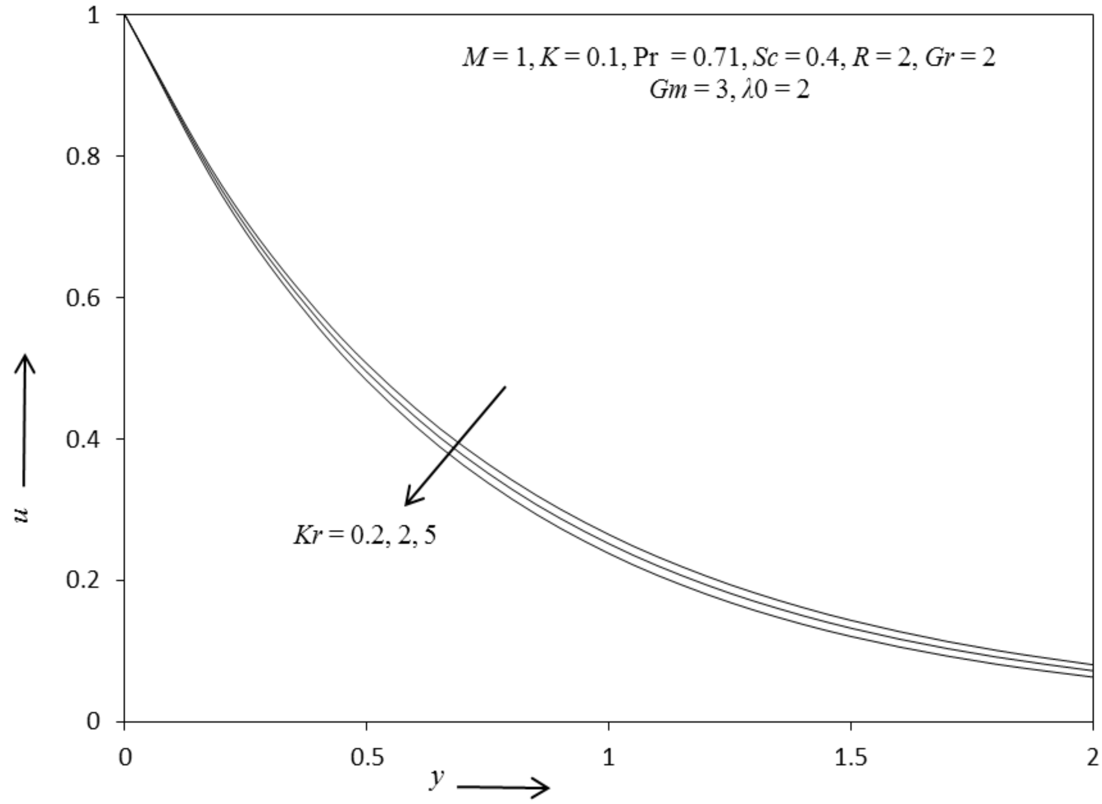


Fig.- (9): Velocity profile of fluid for different values of $Kr$.

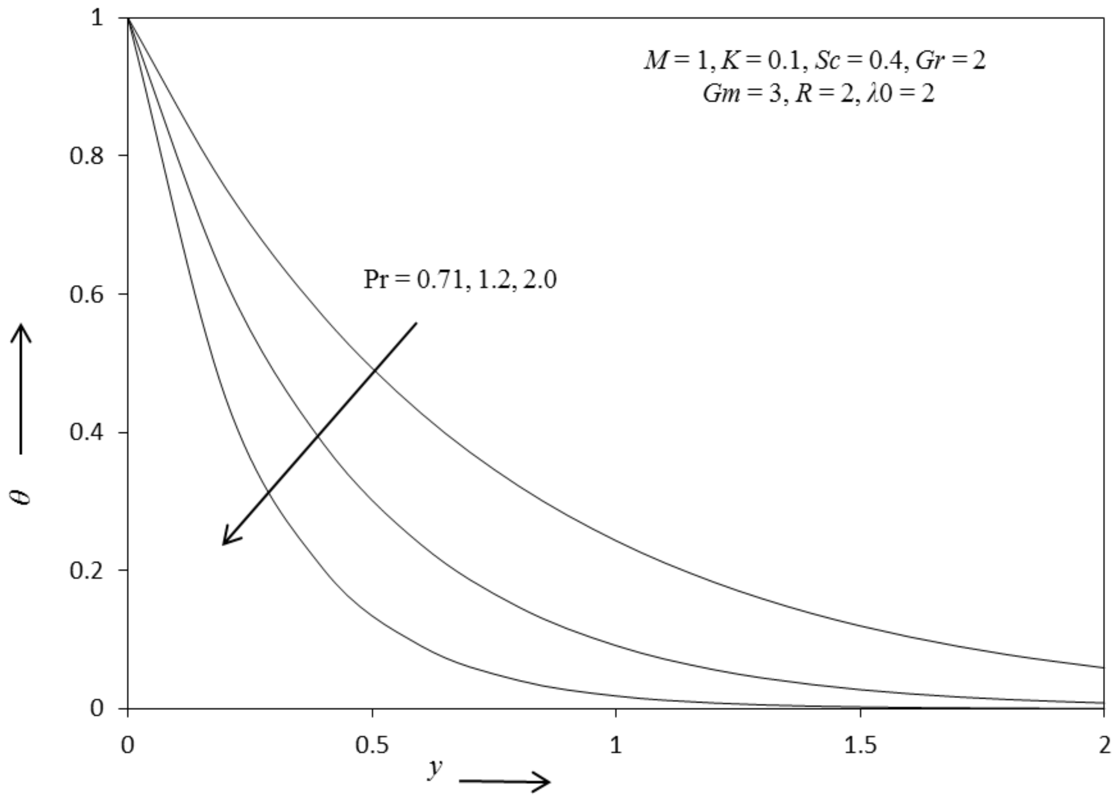


Fig.- (10): Temperature profile of fluid for different values of Pr.

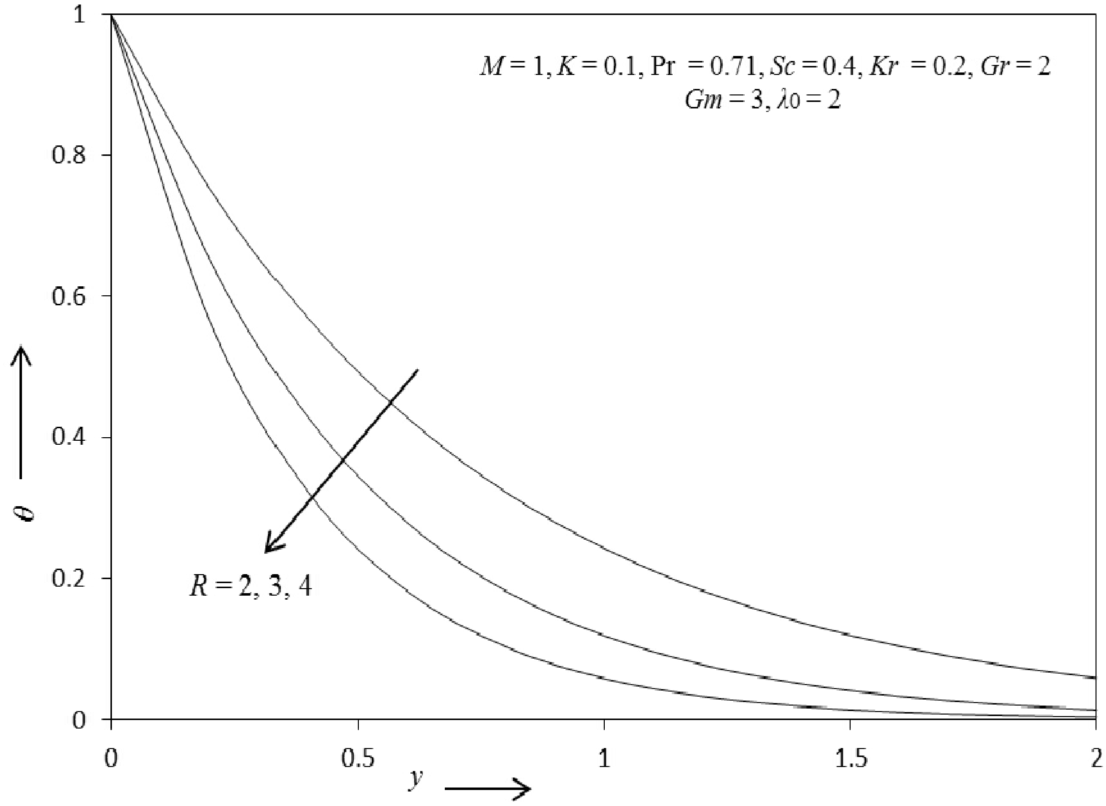


Fig.- (11): Temperature profile of fluid for different values of $R$.

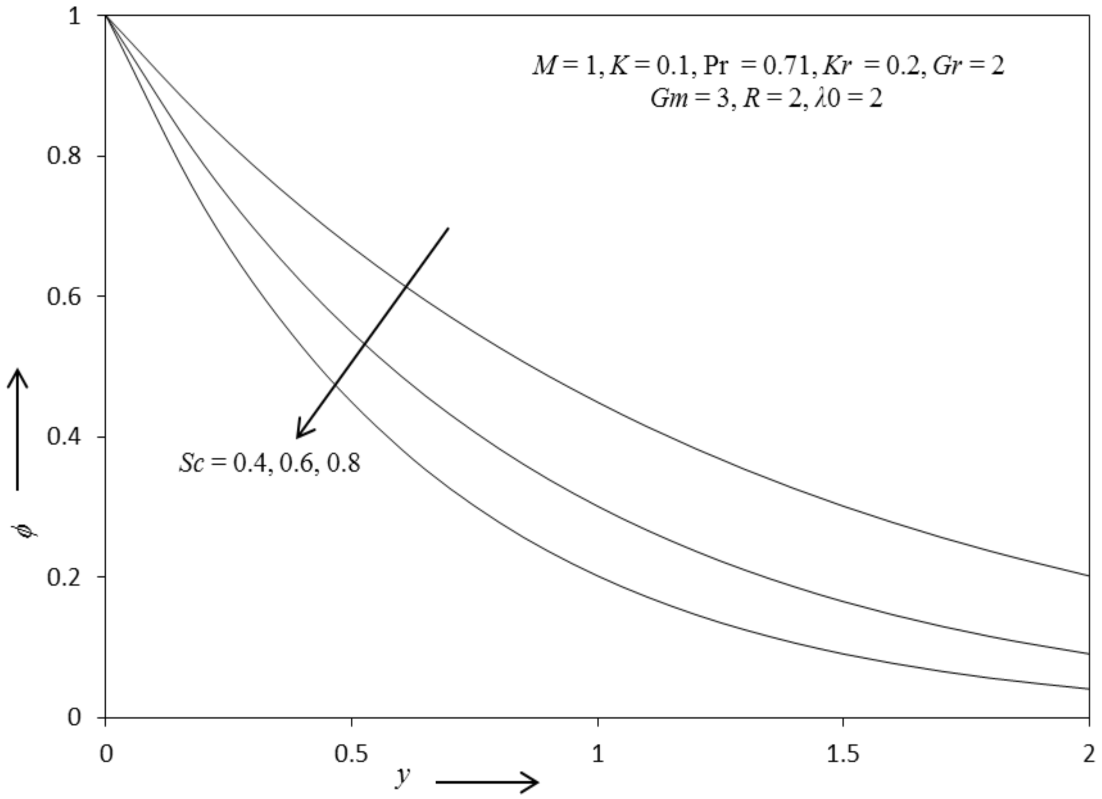


Fig.- (12): Concentration profile of fluid for different values of $Sc$.

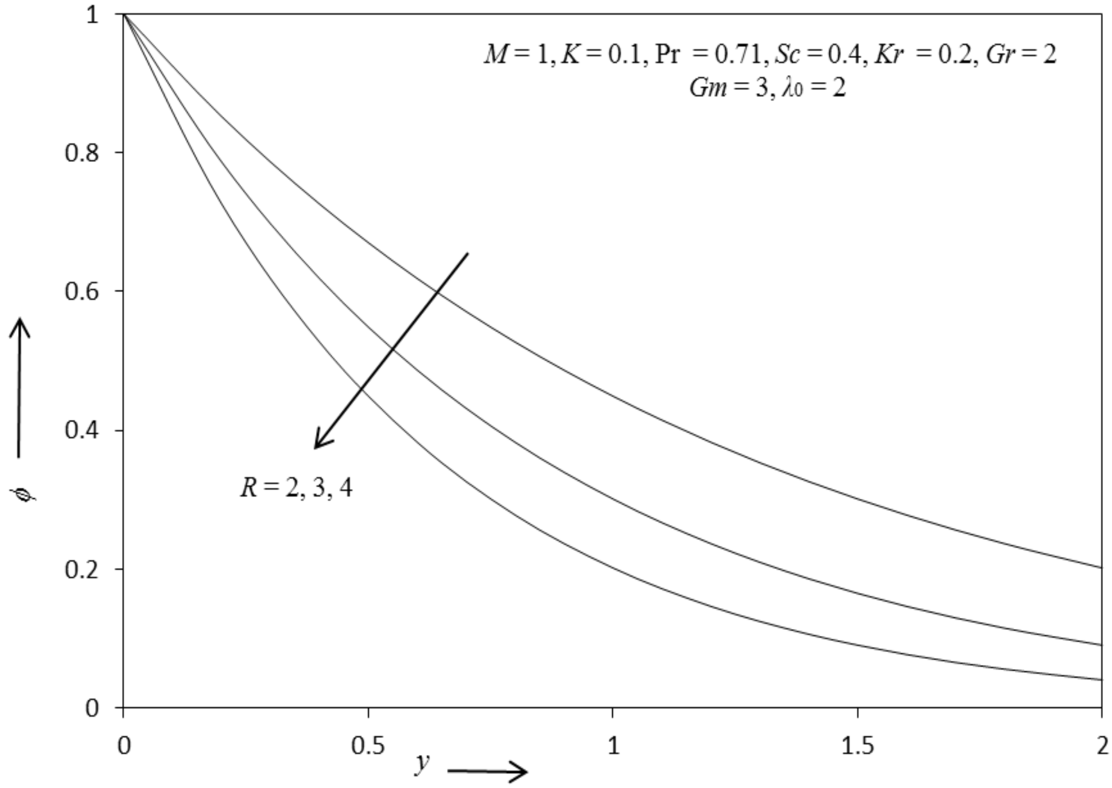


Fig.- (13): Concentration profile of fluid for different values of $R$.

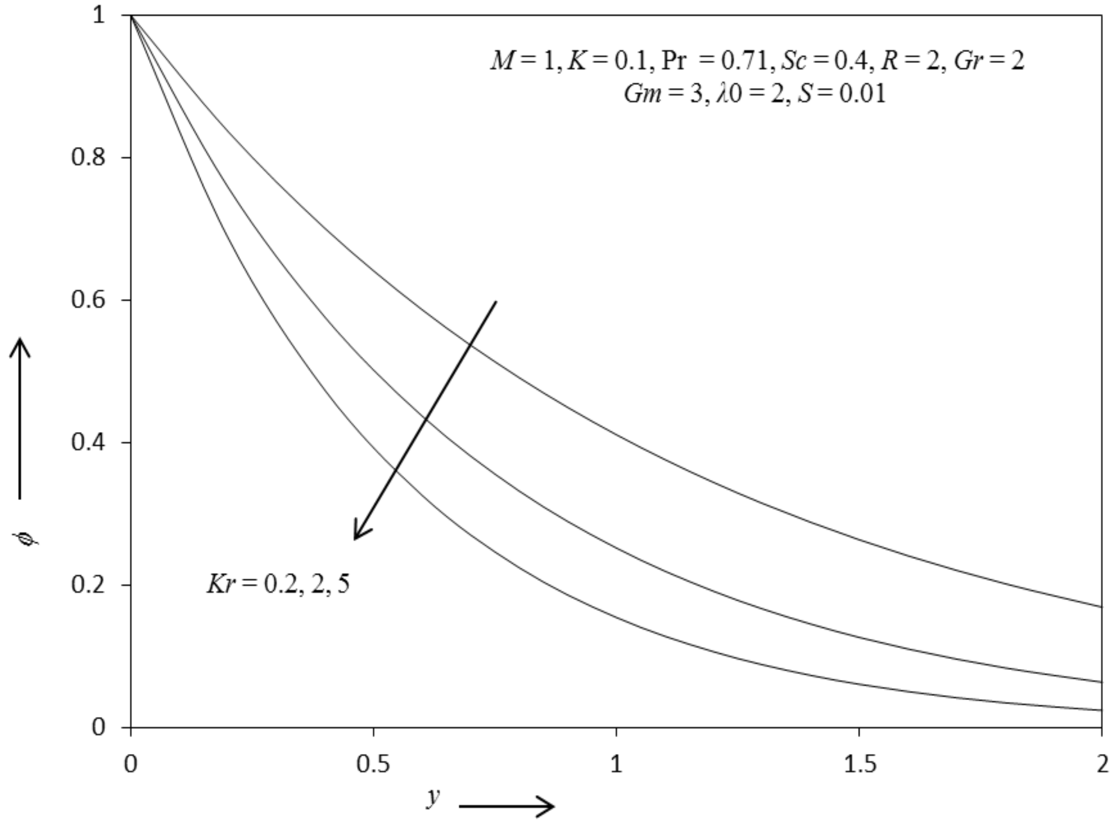


Fig.- (14): Concentration profile of fluid for different values of $Kr$.

## CONCLUSION:

The theoretical solution of the behavior of a first-order homogeneous chemical reaction on free convective boundary layer flow of an electrically conducting visco-elastic, incompressible fluid through porous medium over a continuously moving flat surface in the presence of uniform magnetic field and constant suction is studied. A uniform magnetic field is assumed to be applied perpendicular to the moving flat surface. The governing equations of motion are solved by perturbation technique. The study concludes the following results.

1. The velocity profiles of fluid decreases as $M$ increases.
2. An increase in permeability parameter $K$ causes an increase in velocity profile of boundary layer flow.
3. The velocity of boundary layer flow increases as the Grashof number $Gr$ and modified Grashof number $Gm$ increase.
4. The velocity of boundary layer flow decreases as $R$ increases.
5. The velocity profile of boundary layer flow increases due to increasing visco-elastic parameter ($\lambda_0$).
6. The velocity profile of boundary layer flow decreases as Pr increases.
7. The velocity of boundary layer flow $u$ decreases with increasing in $K_r$.
8. The temperature profile of boundary layer flow decreases as $R$ increases.
9. An increase in Schmidt number $Sc$ leads to decrease in concentration profile of boundary layer flow.
10. An increase in Reynolds number ® causes decrease in concentration profile of boundary layer flow.
11. The concentration profiles of boundary layer flow increases with increasing in $K_r$.

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