Annals of Natural Sciences
(Peer Reviewed and UGC Approved International Journal)
Vol. 4(1), March 2018: 90-93
Journal's URL: http://www.crsdindia.com/ans.html
Email: crsdindia@gmail.com

e-ISSN: 2455-667X

Annals of Natural Sciences

ORIGINAL ARTICLE

A Measurement of Income Inequality Based on Relative Inequalities of Incomes Beyond Lower and Upper Quartiles

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ABSTRACT

Some scholars tried to find measures for income inequality by taking various inequalities into consideration. In the present paper an attempt has been made to develop a measure of income inequality based on relative inequalities of incomes beyond lower and upper quartiles. To show the computational process a hypothetical data has been used.

Key words: *Income Inequality, Relative Inequality, Lower and Upper Quartiles*

Received: 4th Jan. 2018, Revised: 21st Feb. 2018, Accepted: 23rd Feb. 2018

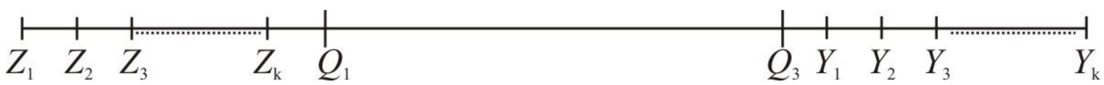
How to cite this article:
Singh D.P. (2018): A Measurement of Income Inequality Based on Relative Inequalities of Incomes Beyond Lower and Upper Quartiles. Annals of Natural Sciences, Vol. 4[1]: March, 2018: 90-93.

INTRODUCTION

In the previous papers Singh (2016, 2017) developed a measure of income inequality by using deviation from maximum income and by using ratio of maximum income to mean income in place of deviation. In the present paper a measure of income inequality based on relative inequalities of incomes beyond lower and upper quartiles has been developed.

PROPOSED MEASURES

Let the income be represented on real line as follows-



Here, the incomes $z_1, z_2, \ldots.., z_k$ are below first quartile (Q_1) and $y_1, y_2, \ldots., y_k$ are the incomes above third quartile (Q_3). Clearly, the incomes $z_1, z_2, \ldots.., z_k$ are usually much smaller than $y_1, y_2, \ldots., y_k$.

Now, consider $y_i / z_j \;\forall\; i,\, j$ as relative individual inequalities.Numbers of such individual inequalities are k^2. Therefore, the average relative inequalities can be considered as a measure of inequality which, mathematically, can be written as:

$$I_{11} = \frac{1}{k^2} \sum_{i=1}^{k} \sum_{j=1}^{k} \frac{y_i}{z_j}$$

$$I_{11} = \left(\frac{1}{k}\sum_{i=1}^{k} y_i\right)\left(\frac{1}{k}\sum_{j=1}^{k} \frac{1}{z_j}\right)$$

$$I_{11} = \frac{\overline{y}}{H.M.(z)}$$

The another relative inequality measure can be defined as:

$$I_{12} = \frac{(y_1.y_2..........y_k)^{1/k}}{(z_1.z_2..........zk)_{1/k}}$$

$$I_{13} = \frac{G.M.(y)}{G.M.(z)}$$

Now instead of taking relative individual inequalities y_i / z_j, we can take $(y_i - z_j)$ as relative inequality.

Now to measure income inequality for a group, as follows-

$$I_{14} = \frac{1}{k^2} \sum_{i=1}^{k} \sum_{j=1}^{k} (y_i - z_j)$$

$$I_{14} = \frac{1}{k^2} \left[\sum_{i=1}^{k} \sum_{j=1}^{k} y_i - \sum_{i=1}^{k} \sum_{j=1}^{k} z_j \right]$$

$$I_{14} = \overline{y} - \overline{z}$$

To make it unit free, the suitable measure is-

$$I_{15} = \frac{\overline{y} - \overline{z}}{\overline{y} + \overline{z}}$$

If all incomes are equal then $I_{15} = 0$ and if $z_1 = z_2 = ...z_k = y_1 = ...y_{k-1} = 0$ $y_k = T$ (Total income), then

$$I_{15} = \frac{T}{k} \Big/ \frac{T}{k} = 1$$

If the simple arithmetic mean is replaced by Geometric mean and Harmonic mean, then the followings are also the measures of income inequality i.e.

$$I_{16} = \frac{G.M.(y_1 ------ y_k) - G.M.(z_1 ------ z_k)}{G.M.(y_1 ------ y_k) + G.M.(z_1 ------ z_k)}$$

$$I_{17} = \frac{H.M.(y_1 ------ y_k) - H.M.(z_1 ------ z_k)}{H.M.(y_1 ------ y_k) + H.M.(z_1 ------ z_k)}$$

Now, to make more sensitive measure of inequality, we can take weighted type index as follows-

$$I_{18} = \alpha \sum_{i=1}^{k} \sum_{j=1}^{k} \left(y_i - z_j\right) w_{ij}$$

Where α is normalizing parameter and w_{ij} as weights.

Now, to determine w_{ij}, we consider

$$w_{ij} \ \alpha \ (y_i - z_j)$$

$$w_{ij} = l\left(y_i - z_j\right)$$

$$\sum_{i=1}^{k} \sum_{j=1}^{k} w_{ij} = l \sum_{i=1}^{k} \sum_{j=1}^{k} \left(y_i - z_j\right)$$

Now take sum of all weights as unity i.e. $\sum_{i=1}^{k} \sum_{i=1}^{k} w_{ij} = 1,$ then

$$1 = l \sum_{i=1}^{k} \sum_{j=1}^{k} \left(y_i - z_j\right)$$

$$l = \frac{1}{k^2 \left(\overline{y} - \overline{z}\right)}$$

$$w_{ij} = (y_i - z_i) / k_2 (\bar{y} - \bar{z})$$

$$I_{18} = \alpha \frac{\sum_{i=1}^{k} \sum_{j=1}^{k} \left(y_i - z_j\right)^2}{k^2 \left(\overline{y} - \overline{z}\right)}$$

Now to determine α, we assume $Z_1 = Z_2 = \ldots\ldots = Z_k = y_1 = y_2 = \ldots\ldots = y_{k\text{-}1} = 0$ and $y = T$ (Total income) and take the index I_{18} as 1 for this special case. Mathematically-

$$1 = \alpha \frac{\sum_{i=1}^{k} \sum_{j=1}^{k} \left(y_i - z_j\right)^2}{k^2 \ T}$$

$$1 = \alpha \frac{\sum_{i=1}^{k} \sum_{j=1}^{k} \left(y_i^2 + z_j^2 - 2 y_i z_j\right)}{k^2 \ T}$$

$$1 = \alpha \frac{\left[kT^2 + 0 - 2T \times 0\right]}{k^2 \ T}$$

$$1 = \alpha \frac{T}{k}$$

$$\alpha = \frac{k}{T}$$

Hence, final measure I_{18} is as follows-

$$I_{18} = \frac{1}{T}\frac{\sum_{i=1}^{k}\sum_{j=1}^{k}\left(y_i - z_j\right)^2}{(\bar{y} - \bar{z})}$$

COMPUTATION FOR A HYPOTHETICAL DATA

Now, we consider the same hypothetical data to compute some proposed measures used in Singh (2016). Suppose there is a firm of 50 persons and the monthly salary of these employees are as follows:

2300	2350	2200	2400	3000	3200
3500	2600	2800	2250	2500	3000
3500	5000	5200	2800	2700	3000
3200	2700	4500	5000	6000	5500
5800	10000	12000	13000	14500	13000
12000	18000	16000	14500	19000	17000
14500	13000	16000	16000	18000	5250
11000	12000	16000	30000	16000	12000
50000	3350				

Now, in order to calculate measures I_{11} to I_{18}, we have

$Q_1 = 3000$ and $Q_3 = 14500$ with k=11

and

	z	y
Mean	2509.091	21090.91
HM	2491.16	18694.78
GM	2500.12	19606.66

T = 50,000

Now, using the above values one may easily calculate various measures as-

I_{11}	I_{13}	I_{14}	I_{15}	I_{16}	I_{17}	I_{18}
8.466299169	7.842286	18581.82	0.787365	0.773814	0.764829	57.81866389

CONCLUSION

In the present paper measure of income inequality has been developed by taking relative inequalities of incomes beyond lower and upper quartiles. Also Now instead of taking relative individual inequalities y_i / z_j measure based by taking (y_i - z_j) as relative inequality has been also discussed.

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