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ORIGINAL ARTICLE

Stochastic Behaviour of a Two Unit Warm Standby System with Preparation Time for Replacement

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ABSTRACT
The present paper deals with the Stochastic Behaviour of a two unit warm standby system with preparation time for replacement, in which after each warm standby unit, the repaired unit is sent for "final trial" with preparation time before sending it for operation. Using regenerative point technique with Markov renewal process, the some of the reliability.
Key words: *Stochastic Behaviour, Warm Standby system and Replacement*

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INTRODUCTION
Several authors including [………………………] working in the field of reliability have analysed various engineering systems by assuming different sets of assumptions. Most of them assumed that after the failure of an operative unit, its repair starts immediately and it continues till its completion without considering the time factor.
But in the real practical situation it is quite reasonable to fix some upper limit of time for completing the repair. If the repairman is able to repair the failed unit within this time period then it is OK. Otherwise the preparation for replacement starts immediately. Once the preparation process is completed the failed unit is sent for replacement by the new one.
Keeping the above view, we in the present chapter analysed a two unit warm standby system with preparation time for replacement. Using regenerative point technique with Markov renewal process, the following reliability characteristics of interest are obtained.
1. Transition and steady state transition probabilities
2. Mean Sojourn times in various states
3. Mean time to system failure (MTSF)
4. Point wise and Steady state availability of the system
5. Expected Busy period of the repairman in (0,t]
6. Expected number of visits by the repairman in (0,t]

MODEL DESCRIPTION AND ASSUMPTIONS
1. The system consists of two non-identical units in which first unit is treated as priority and another as ordinary.
2. Initially first unit is considered to be operative and the second as warm standby.

3. First unit gets priority fore both operation and repair.
4. After failure the unit is sent for repair immediately provided the repair facility is available but in case of first (priority) unit an amount of time has been fixed for repair facility known as "allowed time". If the repair facility is able to complete the repair of the failed unit within this allowed time then it is O.K. otherwise preparation for replacement starts. The repair facility takes a random amount of time in preparation to start replacement.
5. In replacement the failed unit is to be replaced by the new one.
6. The failure time distributions of both the units and preparation time to start replacement are exponential with different parameters while the repair and replacement time distributions are general.

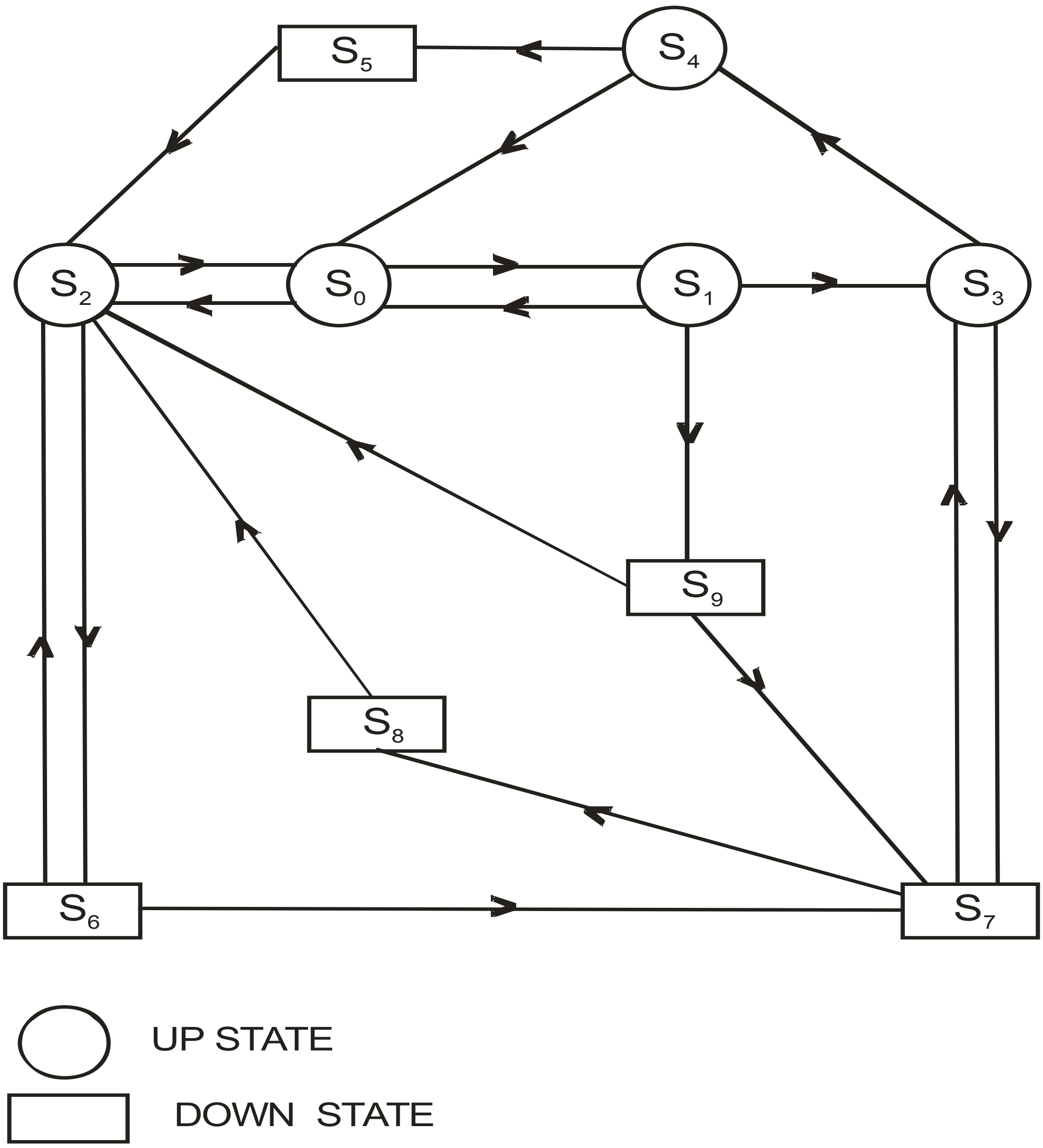

Fig. 1: The transitions between the various states are shown in Fig

NOTATION AND SYMBOLS

N_O : Normal unit kept as operative
N_{WS} : Normal unit kept as warm standby
F_r : Failed unit under repair
F_R : Repair of the failed unit is continued from earlier state
F_{wr} : Failed unit is waiting for repair
F_p : Failed unit under preparation to start replacement
F_P : Preparation is continued form the earlier state
F_{rep} : Failed unit under replacement
F_{REP} : Replacement of the failed unit is continued from earlierstate
α_1 : Constant failure rate of priority unit
α_2 : Constant failure rate of ordinary unit
β : Constant failure rate of warm standby unit
γ : Constant rate of completing preparation
f(.), F(.) : pdf and cdf of time to complete repair of failed priority unit
g(.), G(.) : pdf and cdf of time to complete repair of failed ordinary unit
h(.), H(.) : pdf and cdf of time to complete replacement of the failed unit by the new one
m_1 : Mean time for completing repair
m_2 : Mean time for completing replacement

Using the above notation and symbols the possible states of the system are

<u>Up States</u>

$S_0 \equiv (N_O, N_{WS})$ $S_1 \equiv (F_r, N_O)$ $S_2 \equiv (N_O, F_r)$
$S_3 \equiv (F_p, N_O)$ $S_4 \equiv (F_{rep}, N_O)$

<u>Down States</u>

$S_5 \equiv (F_{REP}, F_{wr})$ $S_6 \equiv (F_r, F_{wr})$ $S_7 \equiv (F_p, F_r)$
$S_8 \equiv (F_{rep}, F_{wr})$ $S_9 \equiv (F_R, F_{wr})$

TRANSITION PROBABILITIES

Let T_0 (=0), $T_1, T_2, ...$ be the epochs at which the system enters the states $S_i \in$ E. Let X_n denotes the state entered at epoch T_{n+1} i.e. just after the transition of T_n. Then $\{T_n, X_n\}$ constitutes a Markov-renewal process with state space E and

$$Q_{ik}(t) = \Pr[X_{n+1} = S_k, T_{n+1} - T_n \leq t \mid X_n = S_i] \quad \text{..........(1)}$$

is semi Markov-Kernal over E. The stochastic matrix of the embedded Markov chain is

$$P = p_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty) \quad \text{..........(2)}$$

By simple probabilistic consideration, the non-zero elements of $Q_{ik}(t)$ are:

$$Q_{01}(t) = {}_0\int^t \alpha_1 e^{-(\alpha_1+\alpha_2)u}\,du = \frac{\alpha_1}{\alpha_1+\alpha_2}[1 - e^{-(\alpha_1+\alpha_2)t}]$$

$$Q_{02}(t) = {}_0\int^t \alpha_2 e^{-(\alpha_1+\alpha_2)u}\,du = \frac{\alpha_2}{\alpha_1+\alpha_2}[1 - e^{-(\alpha_1+\alpha_2)t}]$$

$$Q_{10}(t) = {}_0\int^t e^{-(\beta+\alpha_2)u} f(u)\,du \qquad Q_{13}(t) = {}_0\int^t \beta e^{-(\beta+\alpha_2)u}\,\overline{F}(u)\,du$$

$$Q_{19}(t) = {}_0\int^t \alpha_2 e^{-(\beta+\alpha_2)u}\,\overline{F}(u)\,du$$

$$Q^{(9)}_{12}(t) = {}_0\int^t e^{-\beta u} f(u)\,du - {}_0\int^t e^{-(\beta+\alpha_2)u}\,\overline{F}(u)\,du$$

$$Q^{(9)}_{17}(t) = {}_0\int^t \beta e^{-\beta u}\,du\,\overline{F}(u)\int_u^t \frac{dF(x)}{\overline{F}(u)} - {}_0\int^t e^{-(\beta+\alpha_2)u}\,du\,\overline{F}(u)\int_u^t \frac{dF(x)}{\overline{F}(u)}$$

$Q_{20}(t) = {}_0\int^t e^{-\alpha_1 u} g(u)\, du$ $\qquad Q_{26}(t) = {}_0\int^t \alpha_1 e^{-\alpha_1 u} \overline{G}(u)\, du$

$Q_{34}(t) = {}_0\int^t \gamma e^{-(\gamma+\alpha_2)u}\, du = \frac{\gamma}{\gamma+\alpha_2}[1 - e^{-(\gamma+\alpha_2)t}]$

$Q_{37}(t) = {}_0\int^t \alpha_2 e^{-(\gamma+\alpha_2)u}\, du = \frac{\alpha_2}{\gamma+\alpha_2}[1 - e^{-(\gamma+\alpha_2)t}]$

$Q_{40}(t) = {}_0\int^t e^{-\alpha_2 u} h(u)\, du$ $\qquad Q_{45}(t) = {}_0\int^t \alpha_2 e^{-\alpha_2 u} \overline{H}(u)\, du$

$Q^{(5)}_{42}(t) = {}_0\int^t \alpha_2 e^{-\alpha_2 u}\, du\, \overline{H}(u) \int_u^t \frac{dH(x)}{\overline{H}(u)}$ $\qquad Q_{62}(t) = {}_0\int^t e^{-\beta u} f(u)\, du$

$Q_{67}(t) = {}_0\int^t \beta e^{-\beta u} \overline{F}(u)\, du$ $\qquad Q_{73}(t) = {}_0\int^t e^{-\gamma u} g(u)\, du$

$Q_{78}(t) = {}_0\int^t \gamma e^{-\gamma u} \overline{G}(u)\, du$ $\qquad Q_{82}(t) = {}_0\int^t h(u)\, du$

……….(3-21)

Taking limit as $t \to \infty$, the steady state transition p_{ij} can be obtained from (3-21) . Thus

$$p_{ik} = \lim_{t\to\infty} Q_{ik}(t) \qquad \text{……….(22)}$$

$p_{01} = \frac{\alpha_1}{\alpha_1+\alpha_2}$ $\qquad p_{02} = \frac{\alpha_2}{\alpha_1+\alpha_2}$

$p_{10} = f^*(\beta+\alpha_2)$ $\qquad p_{13} = \frac{\beta}{\beta+\alpha_2}[1 - f^*(\beta+\alpha_2)]$

$p_{19} = \frac{\alpha_2}{\beta+\alpha_2}[1 - f^*(\beta+\alpha_2)]$ $p^{(9)}_{12} = f^*(\beta) - f^*(\beta+\alpha_2)$

$p^{(9)}_{17} = 1 - f^*(\beta) - \frac{\beta}{\beta+\alpha_2}[1 - f^*(\beta+\alpha_2)]$ $p_{20} = g^*(\alpha_1)$

$p_{26} = 1 - g^*(\alpha_1)$ $\qquad p_{34} = \frac{\gamma}{\gamma+\alpha_2}$

$p_{37} = \frac{\alpha_2}{\gamma+\alpha_2}$ $\qquad p_{40} = h^*(\alpha_2)$

$p_{45} = 1 - h^*(\alpha_2) = p^{(5)}_{42}$ $\qquad p_{62} = f^*(\beta)$

$p_{67} = 1 - f^*(\beta)$ $\qquad p_{73} = g^*(\gamma)$

$p_{78} = 1 - g^*(\gamma)$ $\qquad p_{82} = 1$ ……….(23-40)

From the above probabilities the following relation can be easily verifies as;

$p_{01} + p_{02} = 1$ $p_{10} + p_{13} + p_{19} = 1 = p_{10} + p_{13} + p^{(9)}_{12} + p^{(9)}_{17}$

$p_{34} + p_{37} = 1$ $p_{40} + p_{45} = 1 = p_{40} + p^{(5)}_{42}$

$p_{62} + p_{67} = 1$ $\qquad p_{73} + p_{78} = 1$

$p_{82} = 1$ ……….(41-47)

Mean Sojourn times

The mean time taken by the system in a particular state S_i before transiting to any other state is known as mean sojourn time and is defined as

$$\mu_i = {}_0\int^{\infty} P[T>t]\, dt \qquad \text{………(48)}$$

Where T is the time of stay in state S_i by the system.

To calculate mean sojourn time μ_i in state S_i, we assume that so long as the system is in state S_i, it will not transit to any other state. Therefore;

$\mu_0 = \dfrac{1}{\alpha_1+\alpha_2}$ $\quad\mu_1 = \dfrac{1}{\beta+\alpha_2}[1 - f^*(\beta+\alpha_2)]$

$\mu_2 = \dfrac{1}{\alpha_1}[1 - g^*(\alpha_1)]$ $\quad\mu_3 = \dfrac{1}{\gamma+\alpha_2}$

$\mu_4 = \dfrac{1}{\alpha_2}[1 - h^*(\alpha_2)]$ $\quad\mu_6 = \dfrac{1}{\beta}[1 - f^*(\beta)]$

$\mu_7 = \dfrac{1}{\gamma}[1 - g^*(\gamma)]$ $\quad\mu_8 = {}_0\int^\infty \overline{H}(t)\, dt = \mu_5$

$\mu_9 = {}_0\int^\infty \overline{F}(t)\, dt$ …….….(49-57)

CONTRIBUTION TO MEAN SOJOURN TIME

For the contribution to mean sojourn time in state $S_i \in E$ and non-regenerative state occurs, before transiting to $S_j \in E$, i.e.,

$m_{ij} = {}_0\int^\infty t.q_{ij}(t)\, dt = -q'^*_{ij}(0)$ ……………..….(58)

Therefore,

$m_{01} = {}_0\int^\infty t.\alpha_1 e^{-(\alpha_1+\alpha_2)t}\, dt = \dfrac{\alpha_1}{(\alpha_1+\alpha_2)^2}$

$m_{02} = {}_0\int^\infty t.\alpha_2 e^{-(\alpha_1+\alpha_2)t}\, dt = \dfrac{\alpha_2}{(\alpha_1+\alpha_2)^2}$

$m_{10} = {}_0\int^\infty t.e^{-(\beta+\alpha_2)t} f(t)\, dt$

$m_{13} = {}_0\int^\infty t.\beta e^{-(\beta+\alpha_2)t}\, \overline{F}(t)\, dt$

$m_{19} = {}_0\int^\infty t.\alpha_2 e^{-(\beta+\alpha_2)t}\, \overline{F}(t)\, dt$

$m^{(9)}_{12} = {}_0\int^\infty t.e^{-\beta t} f(t)\, dt - {}_0\int^\infty t.e^{-(\beta+\alpha_2)t}\, \overline{F}(t)\, dt$

$m^{(9)}_{17} = {}_0\int^\infty t.\beta e^{-\beta t}\, dt\, \overline{F}(t) \int_u^t \dfrac{dF(x)}{\overline{F}(u)} - {}_0\int^\infty t.e^{-(\beta+\alpha_2)t}\, dt\, \overline{F}(t) \int_u^t \dfrac{dF(x)}{\overline{F}(u)}$

$m_{20} = {}_0\int^\infty t.e^{-\alpha_1 t} g(t)\, dt$ $\quad m_{26} = {}_0\int^\infty t.\alpha_1 e^{-\alpha_1 t}\, \overline{G}(t)\, dt$

$m_{34} = {}_0\int^\infty t.\gamma e^{-(\gamma+\alpha_2)t}\, dt = \dfrac{\gamma}{(\gamma+\alpha_2)^2}$

$m_{37} = {}_0\int^\infty t.\alpha_2 e^{-(\gamma+\alpha_2)t}\, dt = \dfrac{\alpha_2}{(\gamma+\alpha_2)^2}[1 - e^{-(\gamma+\alpha_2)}]$

$m_{40} = {}_0\int^\infty t.e^{-\alpha_2 t} h(t)\, dt$ $\quad m_{45} = {}_0\int^\infty t.\alpha_2 e^{-\alpha_2 t}\, \overline{H}(t)\, dt$

$m^{(5)}_{42} = {}_0\int^\infty t.\alpha_2 e^{-\alpha_2 t}\, dt\, \overline{H}(t) \int_u^t \dfrac{dH(x)}{\overline{H}(u)}$

$m_{62} = {}_0\int^\infty t.e^{-\beta t} f(t)\, dt$ $\quad m_{67} = {}_0\int^\infty t.\beta e^{-\beta t}\, \overline{F}(t)\, dt$

$m_{73} = {}_0\int^\infty t.e^{-\gamma t} g(t)\, dt$ $\quad m_{78} = {}_0\int^\infty t.\gamma e^{-\gamma t}\, \overline{G}(t)\, dt$

$m_{82} = {}_0\int^\infty t.h(t)\, dt$ ……………….(59-77)

By the above expressions, it can be easily verified that

$m_{01} + m_{02} = \mu_0$ $\quad m_{10} + m_{13} + m_{19} = \mu_1$

$m_{10} + m_{13} + m^{(9)}_{12} + m^{(9)}_{17} = {}_0\int^\infty t.f(t)\, dt = m_1$

$m_{20} + m_{26} = \mu_2$ $\quad$ $m_{34} + m_{37} = \mu_3$

$m_{40} + m_{45} = \mu_4$ $\quad$ $m_{40} + m^{(5)}_{42} = {}_0\int^{\infty} t.h(t)\, dt = m_2$

$m_{62} + m_{67} = \mu_6$ $\quad$ $m_{73} + m_{78} = \mu_7$ ……………(78-86)

MEAN TIME TO SYSTEM FAILURE (MTSF)

To obtain the distribution function $\pi_i(t)$ of the time to system failure with starting state S_0.

$\pi_0(t) = Q_{01}(t)\$\pi_1(t) + Q_{02}(t)\$\pi_2(t)$

$\pi_1(t) = Q_{10}(t)\$\pi_0(t) + Q_{13}(t)\$\pi_3(t) + Q^{(9)}_{12}(t)\$\pi_2(t) + Q^{(9)}_{17}(t)$

$\pi_2(t) = Q_{20}(t)\$\pi_0(t) + Q_{26}(t)$

$\pi_3(t) = Q_{34}(t)\$\pi_4(t) + Q_{37}(t)$

$\pi_4(t) = Q_{40}(t)\$\pi_0(t) + Q_{48}(t)\$\pi_8(t)$ …………(87-91)

Taking Laplace Stieltjes transform of relations (87-91), we get

$\tilde{\pi}_0(s) = \tilde{Q}_{01}(s).\tilde{\pi}_1(s) + \tilde{Q}_{02}(s).\tilde{\pi}_2(s)$

$\tilde{\pi}_1(s) = \tilde{Q}_{10}(s).\tilde{\pi}_0(s) + \tilde{Q}_{13}(s).\tilde{\pi}_3(s) + \tilde{Q}^{(9)}_{12}(s).\tilde{\pi}_2(s) + \tilde{Q}^{(9)}_{17}(s)$

$\tilde{\pi}_2(s) = \tilde{Q}_{20}(s).\tilde{\pi}_0(s) + \tilde{Q}_{26}(s)$

$\tilde{\pi}_3(s) = \tilde{Q}_{34}(s).\tilde{\pi}_4(s) + \tilde{Q}_{37}(s)$

$\tilde{\pi}_4(s) = \tilde{Q}_{40}(s).\tilde{\pi}_0(s) + \tilde{Q}_{42}(s).\tilde{\pi}_2(s)$ ………..(92-96)

and solving the above equations (92-96) for $\tilde{\pi}_0(s)$ by omitting the argument 's' for brevity, we get

$\tilde{\pi}_0(s) = N_1(s)/ D_1(s)$ …………(97)

where

$N_1(s) = \tilde{Q}_{01}\tilde{Q}_{17} + \tilde{Q}_{01}\tilde{Q}_{13}\tilde{Q}_{37} + \tilde{Q}_{01}\tilde{Q}^{(9)}_{12}\tilde{Q}_{26} + \tilde{Q}_{01}\tilde{Q}_{13}\tilde{Q}_{34}\tilde{Q}^{(5)}_{42}\tilde{Q}_{26}$

$+ \tilde{Q}_{02}\tilde{Q}_{26}$ …………(98)

and

$D_1(s) = 1 - \tilde{Q}_{01}\tilde{Q}_{10} - \tilde{Q}_{01}\tilde{Q}_{13}\tilde{Q}_{34}\tilde{Q}^{(5)}_{42}\tilde{Q}_{20} - \tilde{Q}_{01}\tilde{Q}^{(9)}_{12}\tilde{Q}_{20} - \tilde{Q}_{02}\tilde{Q}_{20} -$

$\tilde{Q}_{01}\tilde{Q}_{13}\tilde{Q}_{34}\tilde{Q}_{40}$ ………..(99)

By taking the limit s→0 in equation (97), one gets $\tilde{\pi}_0(0) = 1$, which implies that $\tilde{\pi}_0(t)$ is a proper distribution function. Therefore, mean time to system failure when the initial state is S_0, is

$$E(T) = -\frac{d}{ds}\pi_0(s)|_{s=0} = \frac{D'_1(0) - N'_1(0)}{D_1(0)} = N_1/D_1 \qquad \text{..........(100)}$$

where

$$N_1 = \mu_0 + m_1p_{01} + m_2p_{01}p_{13}p_{34} + p_{01}p_{13}\mu_3 + \mu_2(p_{02} + p_{01}p^{(9)}_{12} + p_{01}p_{13}p_{34}) \qquad \text{..........(101)}$$

and

$$D_1 = 1 - p_{01}p_{10} - p_{01}p_{13}p_{34}p^{(5)}_{42} - p_{01}p^{(9)}_{12}p_{20} - p_{02}p_{20} - p_{01}p_{13}p_{34}p_{40} \qquad \text{..........(102)}$$

AVAILABILITY ANALYSIS

System availability is defined as

$A_i(t)$ = Pr[Starting from state S_i the system is available at epoch t without passing through any regenerative state] and

$M_i(t)$ = Pr[Starting from up state S_i the system remains up till epoch t without passing through any regenerative up state]

Thus,

$M_0(t) = e^{-(\alpha_1+\alpha_2)t}$ $\qquad$ $M_1(t) = e^{-(\beta+\alpha_2)t}\,\bar{F}(t)$

$M_2(t) = e^{-\alpha_1 t}\,d.\bar{G}(t)$ $\qquad$ $M_3(t) = e^{-(\beta+\alpha_2)t}$

$M_4(t) = e^{-\alpha_2 t}\,\bar{H}(t)$ ....(103-107)

Now, obtaining $A_i(t)$ by using elementary probability argument;

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q^{(9)}_{12}(t)©A_2(t) + q_{13}(t)©A_3(t) + q^{(9)}_{17}(t)©A_7(t)$

$A_2(t) = M_2(t) + q_{20}(t)©A_0(t) + q_{26}(t)©A_6(t)$

$A_3(t) = M_3(t) + q_{34}(t)©A_4(t) + q_{37}(t)©A_7(t)$

$A_4(t) = M_4(t) + q_{40}(t)©A_0(t) + q^{(5)}_{42}(t)©A_2(t)$

$A_6(t) = q_{62}(t)©A_2(t) + q_{27}(t)©A_7(t)$ $\qquad$ $A_7(t) = q_{73}(t)©A_3(t) + q_{78}(t)©A_8(t)$

$A_8(t) = q_{82}(t)©A_2(t)$ ....(108-115)

Taking Laplace transform of above equation (108-115), we get,

$A^*_0(s) = M^*_0(s) + q^*_{01}(s).A^*_1(s) + q^*_{02}(s).A^*_2(s)$

$A^*_1(s) = M^*_1(s) + q^*_{10}(s).A^*_0(s) + q^{*(9)}_{12}(s).A^*_2(s) + q^*_{13}(s).A^*_3(s) + q^{*(9)}_{17}(s).A^*_7(s)$

$A^*_2(s) = M^*_2(s) + q^*_{20}(s).A^*_0(s) + q^*_{26}(s).A^*_6(s)$

$A^*_3(s) = M^*_3(s) + q^*_{34}(s).A^*_4(s) + q^*_{37}(s).A^*_7(s)$

$A^*_4(s) = M^*_4(s) + q^*_{40}(s).A^*_0(s) + q^{*(5)}_{42}(s).A^*_2(s)$

$A^*_6(s) = q^*_{62}(s).A^*_2(s) + q^*_{27}(s).A^*_7(s)$

$A^*_7(s) = q^*_{73}(s).A^*_3(s) + q^*_{78}(s).A^*_8(s)$

$A^*_8(s) = q^*_{82}(s).A^*_2(s)$ ....(116-123)

Now, solving the equations (116-123) for pointwise availability $A^*_0(s)$, by omitting the arguments ‘s’ for brevity, one gets

$$A^*_0(s) = \frac{N_2(s)}{D_2(s)} \qquad \text{..........(124)}$$

Where

$$N_2(s) = [M^*_0 + q^*_{01}M^*_1 + q^*_{01}q^*_{13}M^*_3 + q^*_{01}q^*_{13}q^*_{34}M^*_4] .[(1 - q^*_{26}q^*_{62})(1 - q^*_{37}q^*_{73}) - q^*_{26}q^*_{67}(q^*_{78}q^*_{82} + q^*_{73}q^*_{34}q^{*(5)}_{42})] + M^*_2[(1 - q^*_{37}q^*_{73})(q^*_{02} + q^*_{01}q^{*(9)}_{12} + q^*_{01}q^*_{13}q^*_{34}q^{*(5)}_{42}) + (q^*_{78}q^*_{82} + q^*_{73}q^*_{34}q^{*(5)}_{42}) .(q^*_{01}q^{*(9)}_{17} + q^*_{01}q^*_{13}q^*_{37})] + [M^*_3q^*_{73} + M^*_4q^*_{73}q^*_{34}] .[q^*_{26}q^*_{67}(q^*_{02} + q^*_{01}q^{*(9)}_{12} + q^*_{01}q^*_{13}q^*_{34}q^{*(5)}_{42}) + (1 - q^*_{26}q^*_{62})(q^*_{01}q^{*(9)}_{17} + q^*_{01}q^*_{13}q^*_{37})] \quad \ldots\ldots\ldots(125)$$

and

$$D_2(s) = [1 - q^*_{01}q^*_{10} - q^*_{01}q^*_{13}q^*_{34}q^*_{40}][(1 - q^*_{26}q^*_{62})(1 - q^*_{37}q^*_{73}) - q^*_{26}q^*_{67}(q^*_{78}q^*_{82} + q^*_{73}q^*_{34}q^{*(5)}_{42})] - q^*_{20}[(1 - q^*_{37}q^*_{73}) .(q^*_{02} + q^*_{01}q^{*(9)}_{12} + q^*_{01}q^*_{13}q^*_{34}q^{*(5)}_{42})] - q^*_{73}q^*_{34}q^*_{40} .[q^*_{26}q^*_{67}(q^*_{02} + q^*_{01}q^{*(9)}_{12} + q^*_{01}q^*_{13}q^*_{34}q^{*(5)}_{42}) + (1 - q^*_{26}q^*_{62})(q^*_{01}q^{*(9)}_{17} + q^*_{01}q^*_{13}q^*_{37})] \quad \ldots\ldots\ldots(126)$$

By taking the limit $s\rightarrow 0$ in the relation (126), one gets the value of $D_2(0) = 0$, therefore the steady state availability of the system when it starts operations from S_0 is

$$A_0(\infty) = \lim_{t\rightarrow\infty} A_0(t) = \lim_{s\rightarrow 0} s.\ A_0^*(s) = N_2(0)/D'_2(0) = N_2/D_2 \quad \ldots\ldots(127)$$

where in terms of

$M^*_0(0) = \mu_0$ $\quad$ $M^*_1(0) = \mu_1$ $\quad$ $M^*_2(0) = \mu_2$

$M^*_3(0) = \mu_3$ $\quad$ $M^*_4(0) = \mu_4$ $\quad$ ....(128-132)

We have,

$$N_2 = [\mu_0 + p_{01}\mu_1 + p_{01}p_{13}\mu_3 + p_{01}p_{13}p_{34}\mu_4][(1 - p_{26}p_{62})(1 - p_{37}p_{73}) - p_{26}p_{67}(p_{78} + p_{73}p_{34}p^{(5)}_{42})] + \mu_2[(1 - p_{37}p_{73})(p_{02} + p_{01}p^{(9)}_{12} + p_{01}p_{13}p_{34}p^{(5)}_{42}) + (p_{78}p_{82} + p_{73}p_{34}p^{(5)}_{42})(p_{01}p^{(9)}_{17} + p_{01}\, p_{13}\, p_{37})] + [\mu_3p_{73} + \mu_4p_{73}p_{34}][p_{26}p_{67}(p_{02} + p_{01}p^{(9)}_{12} + p_{01}p_{13}p_{34}p^{(5)}_{42}) + (1 - p_{26}p_{62})(p_{01}p^{(9)}_{17} + p_{01}p_{13}p_{37})] \quad \ldots\ldots\ldots(133)$$

and

$$D_2 = [\mu_0 + p_{01}m_1 + p_{01}p_{13}\mu_3 + p_{01}p_{13}p_{34}\mu_4][(1 - p_{26}p_{62})(1 - p_{37}p_{73}) - p_{26}p_{67}(p_{78} + p_{73}p_{34}p^{(5)}_{42})] + (\mu_2 + p_{26}\mu_6)[(1 - p_{37}p_{73}) .(p_{02} + p_{01}p^{(9)}_{12} - p_{01}p_{13}p_{34}p^{(5)}_{42}) + p_{01}p_{17}p_{78} + p_{01}p_{13}p_{37}p_{78} + p_{01}p^{(9)}_{17}p_{73}p_{34}p^{(5)}_{42}] + \mu_7[p_{26}p_{67}(p_{02} + p_{01}p^{(9)}_{12} + p_{01}p_{13}p_{34}p^{(5)}_{42}) + (1 - p_{26}p_{62})(p_{01}p^{(9)}_{17} + p_{01}p_{13}p_{37}) \quad \ldots\ldots\ldots(134)$$

BUSY PERIOD ANANLYSIS

Let us define $W_i(t)$ as the probability that the system is under repair/replacement by repair facility in state S_i ε E at time t without transiting to any regenerative state. Therefore

$W_1(t) = \bar{F}(t) = W_6(t)$ $\quad$ $W_2(t) = e^{-\alpha_1 t}\,\bar{G}(t)$

$W_3(t) = e^{-(\gamma+\alpha_2)t}$ $\quad$ $W_4(t) = \bar{H}(t) = W_8(t)$

$W_7(t) = e^{-\gamma t}\,\bar{G}(t)$ $\quad$ ....(135-139)

(i) Now, let $B_i(t)$ is the probability that the system is under repair by repair facility at time t, Thus the following recursive relations among $B_i(t)$'s can be obtained as ;

$B_0(t) = q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t)$

$B_1(t) = W_1(t) + q_{10}(t)©B_0(t) + q^{(9)}_{12}(t)©B_2(t) + q_{13}(t)©B_3(t) + q^{(9)}_{17}(t)©B_7(t)$

$B_2(t) = W_2(t) + q_{20}(t)©B_0(t) + q_{26}(t)©B_6(t)$

$B_3(t) = W_3(t) + q_{34}(t)©B_4(t) + q_{37}(t)©B_7(t)$

$B_4(t) = q_{40}(t)©B_0(t) + q^{(5)}_{42}(t)©B_2(t)$
$B_6(t) = W_6(t) + q_{62}(t)©B_2(t) + q_{27}(t)©B_7(t)$
$B_7(t) = W_6(t) + q_{73}(t)©B_3(t) + q_{78}(t)©B_8(t)$
$B_8(t) = q_{82}(t)©B_2(t)$ ……………(140-147)

Taking Laplace Transform of above equation (140-147), we get,

$B^*_0(s) = q^*_{01}(s).B^*_1(s) + q^*_{02}(s).B^*_2(s)$
$B^*_1(s) = W^*_1(s) + q^*_{10}(s).B^*_0(s) + q^{*(9)}_{12}(s).B^*_2(s) + q^*_{13}(s).B^*_3(s) + q^{*(9)}_{17}(s).B^*_7(s)$
$B^*_2(s) = W^*_2(s) + q^*_{20}(s).B^*_0(s) + q^*_{26}(s).B^*_6(s)$
$B^*_3(s) = W^*_3(s) + q^*_{34}(s).B^*_4(s) + q^*_{37}(s).B^*_7(s)$
$B^*_4(s) = q^*_{40}(s).B^*_0(s) + q^{*(5)}_{42}(s).B^*_2(s)$
$B^*_6(s) = W^*_6(s) + q^*_{62}(s).B^*_2(s) + q^*_{27}(s).B^*_7(s)$
$B^*_7(s) = W^*_7(s) + q^*_{73}(s).B^*_3(s) + q^*_{78}(s).B^*_8(s)$
$B^*_8(s) = q^*_{82}(s).B^*_2(s)$ ……………(148-155)

and solving equations (148-155) for $B^*_0(s)$, by omitting the argument 's' for brevity we get;

$B^*_0(s) = N_3(s)/D_3(s)$ ………………..(156)

Where $D_3(s)$ is same as $D_2(s)$ in (126) and

$$N_3(s) = [W^*_1 + q^*_{01}q^*_{13}W^*_3][(1 - q^*_{26}q^*_{62})(1 - q^*_{37}q^*_{73}) - q^*_{26}q^*_{67}(q^*_{78}q^*_{82} + q^*_{73}q^*_{34}q^{*(5)}_{42})] + (W^*_2 + q^*_{26}W^*_6) .[(q^*_{02} + q^*_{01}q^{*(9)}_{12} + q^*_{01}q^*_{13}q^*_{34}q^{*(5)}_{42})(1 - q^*_{37}q^*_{73}) + (q^*_{78}q^*_{82} + q^*_{73}q^*_{34}q^{*(5)}_{42})(q^*_{01}q^{*(9)}_{17} + q^*_{01}q^*_{13}q^*_{37})] + [W^*_7 + q^*_{73}W^*_3][q^*_{26}q^*_{67}(q^*_{02} + q^*_{01}q^{*(9)}_{12} + q^*_{01}q^*_{13}q^*_{34}q^{*(5)}_{42}) + (1 - q^*_{26}q^*_{62})(q^*_{01}q^{*(9)}_{17} + q^*_{01}q^*_{13}q^*_{37})] \quad \text{…………(157)}$$

In this steady state, the fraction of time for which the repair facility is busy in repair is given by

$$B_0 = \lim_{t \to \infty} B_0(t) = \lim_{s \to 0} s\,B^*(s) = N_3(0)/D'_3(0) = N_3/D_3 \quad \text{………(158)}$$

where D_3 is same as D_2 in (134) and

$$N_3 = [m_1 + p_{01}p_{13}\mu_3][(1 - p_{26}p_{62} - p_{37}p_{73}) - p_{26}p_{67}(p_{78}p_{82} + p_{73}p_{34}p^{(5)}_{42})] + (\mu_2 + p_{26}m_1) .[(p_{02} + p_{01}p^{(9)}_{12} + p_{01}p_{13}p_{34}p^{(5)}_{42})(1 - p_{37}p_{73}) + (p_{78}p_{82} + p_{73}p_{34}p^{(5)}_{42})(p_{01}p^{(9)}_{17} + p_{01}p_{13}p_{37})] + [\mu_7 + p_{73}\mu_3][p_{26}p_{67}(p_{02} + p_{01}p^{(9)}_{12} + p_{01}p_{13}p_{34}p^{(5)}_{42}) + (1 - p_{26}p_{62})(p_{01}p^{(9)}_{17} + p_{01}p_{13}p_{37})] \quad \text{………(159)}$$

(ii) Similarly let $R_i(t)$ is the probability that the system is under replacement by repair facility at time t, Thus the following recursive relations among $R_i(t)$'s can be obtained as ;

$R_0(t) = q_{01}(t)©R_1(t) + q_{02}(t)©R_2(t)$
$R_1(t) = q_{10}(t)©R_0(t) + q^{(9)}_{12}(t)©R_2(t) + q_{13}(t)©R_3(t) + q^{(9)}_{17}(t)©R_7(t)$
$R_2(t) = q_{20}(t)©R_0(t) + q_{26}(t)©R_6(t)$
$R_3(t) = q_{34}(t)©R_4(t) + q_{37}(t)©R_7(t)$
$R_4(t) = W_4(t) + q_{40}(t)©R_0(t) + q^{(5)}_{42}(t)©R_2(t)$
$R_6(t) = q_{62}(t)©R_2(t) + q_{27}(t)©R_7(t)$
$R_7(t) = q_{73}(t)©R_3(t) + q_{78}(t)©R_8(t)$
$R_8(t) = W_8(t) + q_{82}(t)©R_2(t)$ …………….(160-167)

Taking Laplace Transform of above equation (160-167), we get,

$R^*_0(s) = q^*_{01}(s).R^*_1(s) + q^*_{02}(s).R^*_2(s)$
$R^*_1(s) = q^*_{10}(s).R^*_0(s) + q^{*(9)}_{12}(s).R^*_2(s) + q^*_{13}(s).R^*_3(s) + q^{*(9)}_{17}(s).R^*_7(s)$
$R^*_2(s) = q^*_{20}(s).R^*_0(s) + q^*_{26}(s).R^*_6(s)$

$R^*_3(s) = q^*_{34}(s).R^*_4(s) + q^*_{37}(s).R^*_7(s)$
$R^*_4(s) = W^*_4(s) + q^*_{40}(s).R^*_0(s) + q^{*(5)}_{42}(s).R^*_2(s)$
$R^*_6(s) = q^*_{62}(s).R^*_2(s) + q^*_{27}(s).R^*_7(s)$
$R^*_7(s) = q^*_{73}(s).R^*_3(s) + q^*_{78}(s).R^*_8(s)$
$R^*_8(s) = W^*_8(s) + q^*_{82}(s).R^*_2(s)$ ………….(168-175)
and solving equations (168-175) for $R^*_0(s)$, by omitting the argument 's' for brevity we get;
$R^*_0(s) = N_4(s)/D_4(s)$ ………...(176)
Where $D_4(s)$ is same as $D_2(s)$ in (126) and

$$N_4(s) = q^*_{01}q^*_{13}q^*_{34}W^*_4[(1 - q^*_{37}q^*_{73})(1 - q^*_{26}q^*_{62}) - q^*_{26}q^*_{67}q^*_{78} - q^*_{26}q^*_{67}q^*_{73}q^*_{34}q^{*(5)}_{42})] + [q^*_{78}W^*_8 + q^*_{73}q^*_{34}W^*_4][q^*_{26}q^*_{67}(q^*_{02} + q^*_{01}q^{*(9)}_{12} + q^*_{01}q^*_{13}q^*_{34}q^{*(5)}_{42}) + (1 - q^*_{26}q^*_{62})(q^*_{01}q^{*(9)}_{17} + q^*_{01}q^*_{13}q^*_{37})] \quad ………..(177)$$

In this steady state, the fraction of time for which the repair facility is busy in repair is given by

$$R_0 = \lim_{t \to \infty} R_0(t) = \lim_{s \to 0} s\,R^*(s) = N_4(0)/D'_4(0) = N_4/D_4 \quad ……...(178)$$

where D_4 is same as D_2 in (134) and

$$N_4 = p_{01}p_{13}p_{34}m_2[(1 - p_{37}p_{73})(1 - p_{26}p_{62}) - p_{26}p_{67}p_{78} - p_{26}p_{67}p_{73}p_{34}p^{(5)}_{42}) + m_2[p_{78} + p_{73}p_{34}][p_{26}p_{67}(p_{02} + p_{01}p^{(9)}_{12} + p_{01}p_{13}p_{34}p^{(5)}_{42}) + (1 - p_{26}p_{62})(p_{01}p^{(9)}_{17} + p_{01}p_{13}p_{37})] \quad ………(179)$$

EXPECTED NUMBER OF VISITS BY THE REPAIR FACILITY

Let we define, $V_i(t)$ as the expected number of visits by the repair facility in (0,t] given that the system initially started from regenerative state S_i at t=0. Then following recurrence relations among $V_i(t)$'s can be obtained as;
$V_0(t) = Q_{01}(t)\$[1 + V_1(t)] + Q_{02}(t)\$[1 + V_2(t)]$
$V_1(t) = Q_{10}(t)\$V_0(t) + Q^{(9)}_{12}(t)\$V_2(t) + Q_{13}(t)\$V_3(t) + Q^{(9)}_{17}(t)\$V_7(t)$
$V_2(t) = Q_{20}(t)\$V_0(t) + Q_{26}(t)\$V_6(t)$
$V_3(t) = Q_{34}(t)\$V_4(t) + Q_{37}(t)\$V_7(t)$
$V_4(t) = Q_{40}(t)\$V_0(t) + Q^{(5)}_{42}(t)\$V_2(t)$
$V_6(t) = Q_{62}(t)\$V_2(t) + Q_{27}(t)\$V_7(t)$
$V_7(t) = Q_{73}(t)\$V_3(t) + Q_{78}(t)\$V_8(t)$
$V_8(t) = Q_{82}(t)\$V_2(t)$ ………..(180-187)
Taking Laplace stieltjes transform of the above equations (180-187) we get

$$\tilde{V}_0(s) = \tilde{Q}_{01}(s).[1 + \tilde{V}_1(s)] + \tilde{Q}_{02}(s).[1 + \tilde{V}_2(s)]$$

$$\tilde{V}_1(s) = Q_{10}(s).\tilde{V}_0(s) + Q^{(9)}_{12}(s).\tilde{V}_2(s) + Q_{13}(s).\tilde{V}_3(s) + Q^{(9)}_{17}(s).\tilde{V}_7(s)$$

$$\tilde{V}_2(s) = Q_{20}(s).\tilde{V}_0(s) + Q_{26}(s).\tilde{V}_6(s)$$

$$\tilde{V}_3(s) = Q_{34}(s).\tilde{V}_4(s) + Q_{37}(s).\tilde{V}_7(s)$$

$$\tilde{V}_4(s) = Q_{40}(s).\tilde{V}_0(s) + Q^{(5)}_{42}(s).\tilde{V}_2(s)$$

$\tilde{V}_6(s) = Q_{62}(s).\tilde{V}_2(s) + Q_{67}(s).\tilde{V}_7(s)$

$\tilde{V}_7(s) = Q_{73}(s).\tilde{V}_3(s) + Q_{78}(s).\tilde{V}_8(s)$

$\tilde{V}_8(s) = Q_{82}(s).\tilde{V}_2(s)$ ………….(188-195)

and solving the equations (188-195) for $\tilde{V}_0(s)$ by omitting the argument 's' for brevity is

$\tilde{V}_0(s) = N_5(s)/D_5(s)$ …….....(196)

Where

$N_5(s) = (\tilde{Q}_{01} + \tilde{Q}_{02})\{(1 - \tilde{Q}_{26}\tilde{Q}_{62})(1 - \tilde{Q}_{37}\tilde{Q}_{73})$

$- \tilde{Q}_{26}\tilde{Q}_{67}(\tilde{Q}_{78}\tilde{Q}_{82} + \tilde{Q}_{73}\tilde{Q}_{34}\tilde{Q}^{(5)}_{42})\}$ ……..….(197)

and

$D_5(s) = [1 - \tilde{Q}_{01}\tilde{Q}_{10} - \tilde{Q}_{01}\tilde{Q}_{13}\tilde{Q}_{34}\tilde{Q}_{40}][(1 - \tilde{Q}_{26}\tilde{Q}_{62})(1 - \tilde{Q}_{37}\tilde{Q}_{73})$

$- \tilde{Q}_{26}\tilde{Q}_{67}(\tilde{Q}_{78}\tilde{Q}_{82} + \tilde{Q}_{73}\tilde{Q}_{34}\tilde{Q}^{(5)}_{42})] - \tilde{Q}_{20}[(1 - \tilde{Q}_{37}\tilde{Q}_{73})$

$.(\tilde{Q}_{02} + \tilde{Q}_{01}\tilde{Q}^{(9)}_{12} + \tilde{Q}_{01}\tilde{Q}_{13}\tilde{Q}_{34}\tilde{Q}^{(5)}_{42})] - \tilde{Q}_{73}\tilde{Q}_{34}\tilde{Q}_{40}$

$.[\tilde{Q}_{26}\tilde{Q}_{67}(\tilde{Q}_{02} + \tilde{Q}_{01}\tilde{Q}^{(9)}_{12} + \tilde{Q}_{01}\tilde{Q}_{13}\tilde{Q}_{34}\tilde{Q}^{(5)}_{42})$

$+ (1 - \tilde{Q}_{26}\tilde{Q}_{62})(\tilde{Q}_{01}\tilde{Q}^{(9)}_{17} + \tilde{Q}_{01}\tilde{Q}_{13}\tilde{Q}_{37})]$ …………(198)

In steady state the number of visit per unit of time when the system starts after entrance into state S_0 is ;

$V_0 = \lim_{t\to\infty} [V_0(t)/t] = \lim_{s\to 0} s\,\tilde{V}_0(s) = N_5/D_5$ …………(199)

where D_5 is same as D_2 in (134) and

$N_5 = (1 - p_{26}p_{62})(1 - p_{37}p_{73}) - p_{26}p_{67}p_{72} - p_{26}p_{67}p_{73}p_{34}p^{(5)}_{42}$ ………..(200)

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